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NEW RESEARCHES ON THE MAGNETIZATION OF FERRO-MAGNETIC SUBSTANCES BY ROTATION AND THE NATURE OF THE ELEMENTARY MAGNET.

By S. J. Barnett and L. J. H. Barnett.

NEW RESEARCHES ON THE MAGNETIZATION OF FERROMAGNETIC SUBSTANCES BY ROTATION AND THE NATURE OF THE ELEMENTARY MAGNET.[1]

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1. **Theoretical relations.** In earlier papers [2] it has been shown that the rotation of iron, cobalt or nickel about a fixed axis in a magnetically neutral region at an angular velocity N revolutions per second magnetizes the substance as it would be magnetized, at rest, by the application of an axial magnetic field with intensity

$$H_r = \lambda N \tag{1}$$

where λ, within the limits of the experimental error, is constant for any one substance and identical for all the substances investigated. The quantity λN may be called the *intrinsic magnetic intensity of rotation.*

Furthermore, on the assumption that only one kind of elementary magnet, or magneton, is responsible for the magnetism of a substance, classical dynamical theory has been shown to require that λ be equal to R, the ratio of the angular momentum of the magneton to its magnetic moment, multiplied by 2π. Thus we have, on this assumption,

$$H_r = \lambda N = 2\pi R N \tag{2}$$

For a magneton such as has ordinarily been held responsible for magnetism, and consisting of an electron or a ring of electrons, each with mass m and charge e, in revolution about a fixed positive nucleus, classical theory gives

$$2\pi R = 2\pi \times 2\, m/e \tag{3}$$

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[1] Read before the American Physical Society, April 25, 1924, and before the American Academy of Arts and Sciences, May 14, 1924. Revised, June, 1925.

[2] S. J. Barnett, *Phys. Rev.*, **6**, 239, 1915, and **10**, 7, 1917. The fundamental idea on which these experiments are based occurred to me in 1909 (*Science*, **30**, 413, 1909), while thinking about the origin of solar and terrestrial magnetism. In 1918 I found that John Perry, with the same fundamental idea in mind (*Spinning Tops*, October, 1890, footnote on p. 112, a book which I had read more than twenty years before), had tried, but without success, to detect a change in the magnetization of an iron rod produced by its rotation. If, as is very probable, I had read Perry's footnote, it had long been forgotten.— S. J. B.

The quantity m/e for an electron has been determined with considerable precision in well-known experiments. Substituting the experimental value for slow motion in (3) we obtain

$$2\pi R = -7.10 \times 10^{-7} \text{ gauss/rev. per second} \qquad (4)$$

The values of λ which we have obtained from experiment agree in sign with this value of $2\pi R$, but they are much smaller in magnitude. They therefore prove either (1) that in producing ferromagnetic phenomena positive magnetons are associated with negative magnetons, whose effect is preponderant; or (2) that the negative magneton is not an orbital electron or a ring of electrons, but a magneton with a much smaller value of R; or else (3), that the classical theory according to which equation (2) is derived is not valid, or (4) that equation (3) is not valid, or both.

Our phenomenon is undoubtedly connected closely with the Zeeman effect, as our magnetons may be considered to be executing regular precession with frequency N under the action of the torques upon them brought about by the rotation. This consideration leads at once to equation (2), which is equivalent to the classical equation for the normal Zeeman effect.[3] As Landé has suggested, the anomaly in the Zeeman effect, which Sommerfeld and Debye had partially explained by the ideas of spatial quantization (now supported in the field of magnetism by the work of Pauli,[4] Sommerfeld,[5] Epstein,[6] Gerlach,[7] and Gerlach and Stern[8]), is probably related closely to the anomaly in our phenomenon. This anomaly Landé [9] and Sommerfeld [10] have attempted to explain by a process which appears to be equivalent to identifying our magneton with the atom in the s-state and attributing to this a value of R equal to m/e, which is approximately the value of R given by our experiments.

2. **General experimental methods.** In the experimental study of magnetization by rotation two general types of methods may be

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3 S. J. Barnett, N. R. C. Bulletin on *Theories of Magnetism*, German translation, 1925.
4 W. Pauli, Jr., *Phys. Zeit.*, **21**, 615, 1920.
5 A. Sommerfeld, *Ann. d. Phys.*, **70**, 32, 1923; *Zeit. f. Phys.*, **19**, 221, 1923.
6 P. S. Epstein, *Science*, **57**, 532, 1923.
7 W. Gerlach, *Phys. Zeit.*, **24**, 275, 1923.
8 Gerlach and Stern, *Ann. d. Phys.*, **74**, 673, 1924; W. Gerlach and A. C. Cilliers, *Z. f. Phys.*, **26**, 106, 1924; W. Gerlach, *Ann. d. Phys.*, **76**, 160, 1925.
9 A. Landé, *Phys. Zeit.*, **24**, 44, 1923.
10 A. Sommerfeld, *Ann. d. Phys.*, **73**, 209, 1924; Atombau, p. 635, 1924.

used, viz., methods of electromagnetic induction and magnetometer methods.

In all methods the substance under investigation is preferably in the form of an approximately cylindrical rod, or *rotor*, which is mounted with its axis horizontal and in the magnetic prime vertical in a region in which the earth's magnetic field has been neutralized as nearly as practicable.

In the methods of electromagnetic induction the intrinsic magnetic intensity of rotation λN is determined by comparing the change of flux through the rotor, due to rotation about its axis at measured speed, with the change of flux produced through the same rotor by the application, parallel to the axis of rotation, of a uniform magnetic field of known intensity. The changes of flux are proportional to the intensities, if small.

In the only induction method which we have used with success the changes of flux were measured ballistically, with a galvanometer of the type which has come to be known as a fluxmeter. Two nearly similar rods *A* and *B* of steel (Fig. 1) were mounted parallel, and two similar

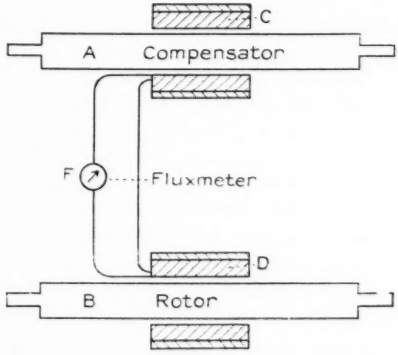

FIG. 1.

coils *C* and *D* of insulated wire were mounted about their centers. The coils, at rest, were connected in series with one another and with the fluxmeter *F*, and in opposition, so that the variations in the earth's field might produce no effect on the fluxmeter. One of the two rods served as rotor; the other was called the *compensator*.

In another method,[11] which has not been used, and which we believe would involve still more serious difficulties, the rod may be oscillated about its axis instead of being set into continuous unidirectional rotation; and the alternating electromotive force thus developed in the surrounding coil of wire may be compared with the electromotive force produced by an alternating field of known intensity, an amplifier being used to increase the sensibility if necessary.

Earlier induction methods were not successful. In one series of experiments the magnetic circuit was almost wholly of iron. The cylinder under investigation rotated between the pole faces of a large U-shaped electromagnet, with minute air gaps; and similar coils on the two legs of the iron core, connected in series in the usual manner, were in series with the fluxmeter. But relatively large extraneous reactions, too complicated to interpret,[12] masked the small effect under investigation. In a still earlier series, the method was essentially similar to that which was later successful, but here also large extraneous disturbances,[13] too complicated to interpret, masked the effect looked for.

In the magnetometer method an astatic magnetometer is mounted with the center of its lower magnet system in the polar axis (magnetometer in *axial position*), or, preferably, in the equatorial plane (magnetometer in *equatorial position*) of the rod, or *rotor*, under investigation. A second rod, the *compensator*, is so mounted as to have as nearly as practicable the same relation to the upper magnet system as the rotor has to the lower, so as to eliminate in a measure disturbances due to variations in the earth's field. The magnetometer deflection produced by reversing the direction of rotation of the rotor, driven at a measured speed, is compared with the deflection produced by reversing a uniform magnetic field of known intensity in the rotor parallel to its axis. The deflections are proportional to the changes in the magnetic moments in the two cases, and these are proportional to the intrinsic intensity of rotation and the standardizing field intensity.

3. **Early successful experiments.** Our first conclusive experiments were made in 1914 at the Ohio State University on cold-rolled

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11 Suggested by an investigation of Tolman's in a different field, now published. See. R. C. Tolman, S. Karrer, and E. W. Guernsey, *Phys Rev.*, **21**, 525, 1923.

12 One of these reactions, as suggested by Dr. A. E. Flowers, was doubtless due to the nonuniformity of the residual axial field through the rotor, or, rather, to its departure from axial symmetry.

13 Professor C. E. Mendenhall suggested torsion, but was inclined to doubt the applicability of the suggestion when he learned that the rod was 7 cm. in diameter.

steel by the fluxmeter method of electromagnetic induction, and were described before the Ohio Academy of Sciences and the American Physical Society in November and December of that year.[14]

They gave for λ the value -3.6×10^{-7} gauss/r.p.s. Another and better investigation by the same general method in 1915[14] gave $\lambda = -(3.1+) \times 10^{-7}$ gauss/r.p.s.; or -3.4×10^{-7} gauss/r.p.s. if we consider only observations at the highest speed, which there is much reason to consider the most trustworthy. All these values are equal to $\frac{1}{2} \times 4\pi m/e$ within the experimental errors.

These experiments were followed by an investigation[15] made in the same laboratory and completed in 1917 on cold-rolled steel, nickel, and cobalt. The values of λ obtained were negative as before, but were intermediate between $\frac{1}{2} \times 4\pi m/e$ and $4\pi m/e$. We also made, a little later, preliminary experiments, consistent with the others, but much rougher, on soft iron and Heusler's alloy.[16]

In the meantime, early in 1915, but *after our first conclusive experiments were described to the American Physical Society*, Einstein and de Haas[17] published experiments on the converse effect, viz., rotation by magnetization, in iron, which yielded, when interpreted on the simplest hypothesis, $\lambda = 4\pi m/e$, though *the sign of the effect was not determined with certainty*.[18] This was done later in the same year, and in 1916, by de Haas[19] and Einstein[20] separately. In 1918 a much more thorough investigation of this converse effect, in both iron and nickel, was published by J. Q. Stewart,[21] who obtained values of λ similar to those we had obtained in 1914 and again in 1915, and equal to one half the value obtained by Einstein and de Haas.

We here call special attention to the priority of our 1914 work over all related work by others, because this work has frequently and incorrectly been referred to[22] as if it had followed, or had been presented to the scientific world later than, or was a (later) confirmation of the work of Einstein and de Haas.

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14 S. J. Barnett, *Phys. Rev.*, **6**, 239, 1915.
15 S. J. Barnett, *Phys. Rev.*, **10**, 7, 1917.
16 S. J. Barnett, *Science*, August 7, 1918.
17 A. Einstein and W. J. de Haas, *Verh. d. D. Phys. Ges.*, **17**, 152, 1915.
18 As pointed out by Lorentz. See A. Einstein, *Verh. d. D. Phys. Ges.*, **17**, 203, 1915; also W. J. de Haas, *K. Akad. Amsterdam Proc.*, **18**, 1280, 1916.
19 W. J. de Haas, *loc. cit.*
20 A. Einstein, *Verh. d. D. Phys. Ges.*, **18**, 173, 1916.
21 J. Q. Stewart, *Phys. Rev.*, **11**, 100, 1918.
22 H. S. Uhler, *Am. J. Sci.*, **41**, 558, 1916; B. Cabrera, *J. de chim. Phys.*, **16**, 455, 1918; R. A. Millikan, *The Electron*, 2d ed., p. 212, 1924 (also 1st ed.); A. Sommerfeld, *Atombau*, 4th ed., p. 635, 1924; *and numerous others.*

4. **New experiments.** The disagreement between our values of λ and the commonly accepted value of $2\pi R$ for electron orbits, and the discrepancies between the values obtained in the different investigations, together with the great importance which must be attached to a more precise knowledge of the magnitude of the constant λ for the substances already investigated and to its determination for other ferromagnetic substances, led us to begin in 1918 a more elaborate investigation of magnetization by rotation by the magnetometer method. In that year we removed to Washington, and were able to do but little more with the work before the summer of 1920. Since that time, as opportunity has offered, and especially in the years 1922–23, we have made an extensive investigation of many ferromagnetic substances. All this work is in agreement, within the limits of the accidental errors, with the fluxmeter observations of 1914 and 1915, and shows that the systematic errors were not completely removed in the earlier magnetometer work.[23]

The rotors, compensating frame, magnetometer, and other appliances constructed for this work in Ohio, and for the most part used in the earlier investigations, were kindly loaned to us by the Ohio State University. With such modifications as were found desirable most of them were used, and rendered important services, in the new work.

5. **The laboratory and its magnetic field.** Nearly all of the new rotation experiments were carried on in a small concrete laboratory,[24] forming a part of the plant of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, and located at a considerable distance from the main building, from which power was obtained. This laboratory is constructed of practically nonmagnetic material throughout. It consists of a single room with floor space 25×50 feet, the longer sides being nearly parallel to the magnetic meridian. The concrete floor, on which the piers carrying the rotation apparatus were mounted, is twelve inches thick.

A survey of the undisturbed magnetic field of the laboratory was made in 1920 by Messrs. W. F. Wallis and A. Sterling, and showed that the field is very nearly uniform, the departure from uniformity being entirely negligible for the purposes of this investigation. For the sake of the experimental work it was, of course, necessary to intro-

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[23] For brief accounts of this work up to April, 1922, see S. J. Barnett and L. J. H. Barnett, *Proc. American Phys. Soc.* for December, 1920 (*Phys. Rev.*, **17**, 404, 1921) and for April, 1922 (*Phys. Rev.*, **20**, 90, 1922).

[24] See J. A. Fleming, *Researches of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington*, **4**, 351, 1921.

duce into the building many instruments with considerable magnetic moments, but they were so placed, or so oriented, sometimes in conjunction with permanent magnets introduced for the purpose, as to produce negligible effects in the region occupied by the rotating body.

Also, to keep the field in the region about the rotor as nearly uniform as possible, the greatest care was taken in the selection and testing of the materials used for the construction of the adjacent apparatus and piers, with the result that they were certainly so free from iron as to have no appreciable effects. We are greatly indebted to the Bureau of Standards for supplying us with a considerable quantity of particularly pure sand for our piers, and for the use of their magnetic separator in improving the rest of the sand which was used. Many of the non-magnetic castings were made in the foundry belonging to our laboratory in Washington.

The fluctuations of the magnetic field. Night Work. In this work, as in the earlier magnetometer work, it was necessary to make the principal experiments after one o'clock at night in order to avoid as far as possible disturbances due to fluctuations of the earth's magnetic field.

The disturbances due to the electric railways seemed to us to become worse as the time progressed, and we finally found it advisable to restrict the rotation observations almost entirely to the interval between half past two and half past four, or even a quarter after four, in the morning. Shortly after we began the next to the last series (Series A) of observations, the Capital Traction Co. started to operate cars on its Chevy Chase line, five-eighths of a mile from our laboratory, throughout the interval in which we worked. When, however, Mr. John Hanna, Vice-President, in charge of operations, learned of the serious interference this service made with our work, the electric cars were most kindly taken off and a bus service substituted. For this generous interest in our work we are greatly indebted to Mr. Hanna and the Company.

Solar disturbances, as would be expected, ordinarily gave little trouble at night. Moreover, the last and most important series of observations (Series B) was made at the time of the sun-spot minimum. For this series the average extreme changes in a single night, from two o'clock to five o'clock A.M., in the hourly means of the horizontal and vertical intensities, as calculated from the records of the Cheltenham Observatory, kindly furnished us by the Coast and Geodetic Survey, were only about 1 part in 5500 and 1 part in 35,000, respectively — quantities beyond our compensation requirements.

6. **The magnetometers.** Two magnetometer cases, one of them modified from the Ohio instrument and both constructed in much the same way, together with several suspension systems, were used in the course of the work.

The suspension, illustrated in Figure 2, was in four parts. The uppermost part, *A*, was a fine quartz fiber, whose substitution for silk considerably improved the behavior of the instrument. The fiber was attached by shellac to the torsion head above and to a geometric hook below. This engaged with a second geometric hook at the top of an aluminum rod *B*, which carried an aluminum damping vane *C* and the mirror *D* and ended below in a small geometric hook *E*. This hook carried the support *F* of the upper magnet system *G*. The support *F* was an aluminum rod fashioned into a geometric hook at its lower end and bent into a small loop with angular top and vertical sides at the upper end. The geometric hook at the bottom of *F* carried a third aluminum rod *I*, terminating above and below in geometric hooks. The lower hook carried the support *J* of the lower magnet system *K*, which was another aluminum rod fashioned into a geometric hook at the top and a loop with two sides vertical at the bottom. The distance between the centers of the two systems of magnets was about 48 cm.

There are three reasons for dividing the aluminum rod carrying the mirror and magnets into three pieces: In the first place, the shorter pieces could be made much more nearly straight and coaxial when mounted together than a single piece of the same total length; in the second place, the intermediate piece, without magnets, could be handled more safely than the others when a twist had to be produced to bring the two magnet systems into parallelism; and in the third place it was desirable for some other experimental work to have the magnet systems closer together than they were in the rotation experiments. This can be accomplished by inverting the central piece.

In all but the earlier part of the work each magnet system consisted of six or three rods about 5 mm. long, 0.17 mm. thick, and 0.35 mm. wide. They were of hardened steel and were kindly provided by Mr. Olof Ohlson, Technical Director of the Waltham Watch Company, at our request for steel treated in such a way as to make the mechanical properties as nearly permanent as possible (according to the recommendations of the Bureau of Standards). By means of a special frame the rods of each system were mounted parallel and central on the appropriate aluminum loop normal to its vertical sides and equidistant. All but one of the rods in each system were fastened per-

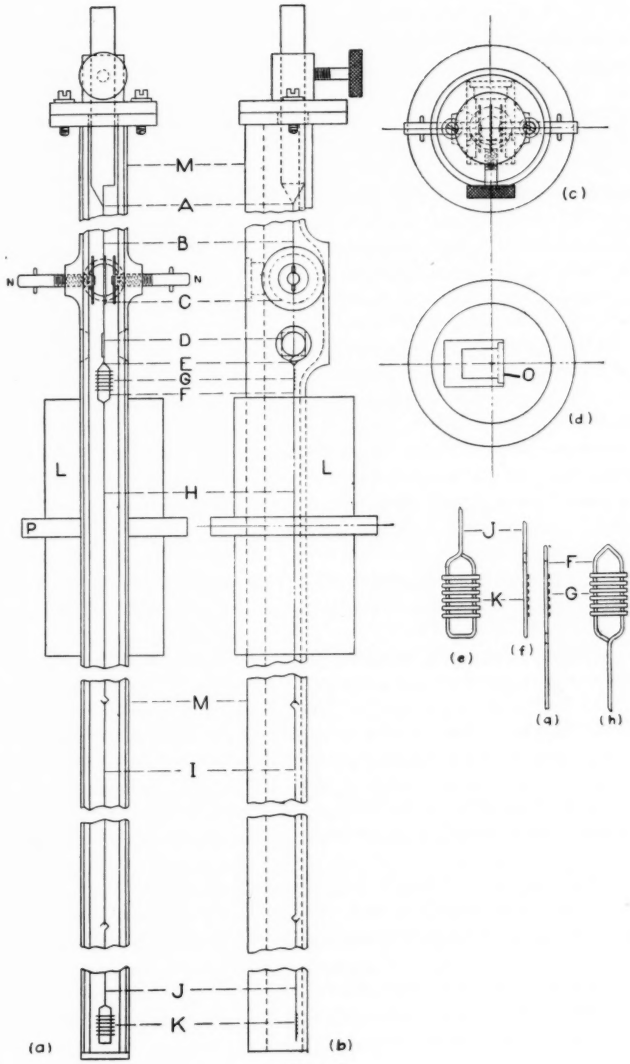

FIG. 2.

manently in place with shellac dissolved in absolute alcohol. The remaining rod was attached with a minute quantity of soft wax. The two systems were then magnetized in an intense magnetic field, and their moments were compared by means of a sensitive astatic magnetometer.

It was our practice in earlier work, in case the moments did not agree within a sufficiently small fraction of one per cent, to remove the waxed-on magnet from the system having the greater moment and reduce its length on an oil stone, then to replace it, remagnetize both systems, and test again, this process being kept up as long as necessary. The two waxed-on magnets were then fastened permanently with shellac, which was allowed to dry without the application of artificial heat. The mechanical treatment, however, changed the properties of the ground magnet, and was abandoned as unsatisfactory. In the case of one suspension it happened that the two systems, whose moments on the first comparison differed by more than 3 per cent, acquired the same moment within about 0.3 per cent when the two removable magnets were interchanged and remagnetized. This system became better with time, and then retained its equality of moments within a small fraction of one per cent for more than two years. This magnetometer system we continued to use for several weeks in the beginning of the last series of observations. Then it was replaced by a system with three-magnet magnets, whose moments also were equal, and remained equal as long as tested in Washington, within a small fraction of one per cent.

Another method, much better than that of simple trial and error, was used in constructing some systems near the end of the investigation. One reason for mounting the magnets equidistant was to make the demagnetizing factors the same for the two systems. It finally occurred to Mr. Kotterman, our assistant, who made most of these systems for us, to use this principle to adjust the moments of the systems to equality. This is very easily done by simply moving one or both of the loose magnets to or from the others to decrease or increase the moment.

To adjust the two magnets to parallelism, the final method was first to hang the magnet system, in three sections, from a proper support, with a long straight aluminum wire fastened horizontally to one side of each loop. Then, if necessary, the central section (or one of the other sections) was twisted until the two horizontal wires became parallel within a fraction of a degree, the sections being separated, of course, while the twisting was done.

When the magnets are geometrically parallel, however, the magnetic axes are not necessarily parallel, and the angle between their axes had to be tested electrically, by processes discussed below in connection with other methods of testing the magnetometer, without removing the suspension from the case.

After the system was properly magnetized and astaticized the three sections together were hung from the mirror support and (usually) cemented together with minute quantities of shellac.

The moment of each of the six-magnet systems was about 0.4 c.g.s. electromagnetic unit.

The magnetometer mirrors were plane, some round and some square, and about 6 mm. along a diameter or edge. One of the mirror supports had two mirrors, facing in opposite directions, so that the magnetometer could be reversed without altering the position of the reading appliances.

The magnetometer case, in which the suspension was mounted by means of a simple torsion head, is also shown in Figure 2. The main part of the case was of soft nonmagnetic brass. It was cast in two (or three) pieces, the short thick portion L being soldered on after the long thin part (or parts), M, had been grooved to hold the suspension. In the original apparatus the supports NN of the damping plates were made to slide in tubes for adjustment, but in the magnetometers used in the present work the plate supports and the small protrusions (or castings) by which they were mounted on the case were threaded, so that excellent adjustment could be made much more readily. The case was made entirely open on one side, but could be closed by long narrow brass strips O pressed against narrow ledges milled for the purpose. Holes cut for the mirror were covered with pieces of spectacle lenses, and holes cut opposite the ends of a horizontal diameter of the damping vane covered with plane glass or mica. A small copper disc covered with a small quantity of radioactive salt was, in the later work, placed at the bottom of the case, which was sealed with soft wax.

The part L of the case was designed to be cylindrical (except for a protruding flange P) and coaxial with the suspension. The lower part of the cylinder was intended to fit accurately a cylindrical hole in the magnetometer support, A, Figure 3, to which the case could be secured by means of two (and later four) small clamps acting on the flange, P (Figures 2 and 4). These conditions were approximately fulfilled, and the magnetometer could be turned about the vertical and clamped in any azimuth with little if any readjustment.

To read the magnetometer deflections, the image of the filament of a

General Electric 10-volt 35-watt nitrogen-filled galvanometer lamp formed by the mirror and suitable lenses was focussed on a translucent scale about 6.2 m. from the instrument. The position of one edge was read to 0.1 mm.

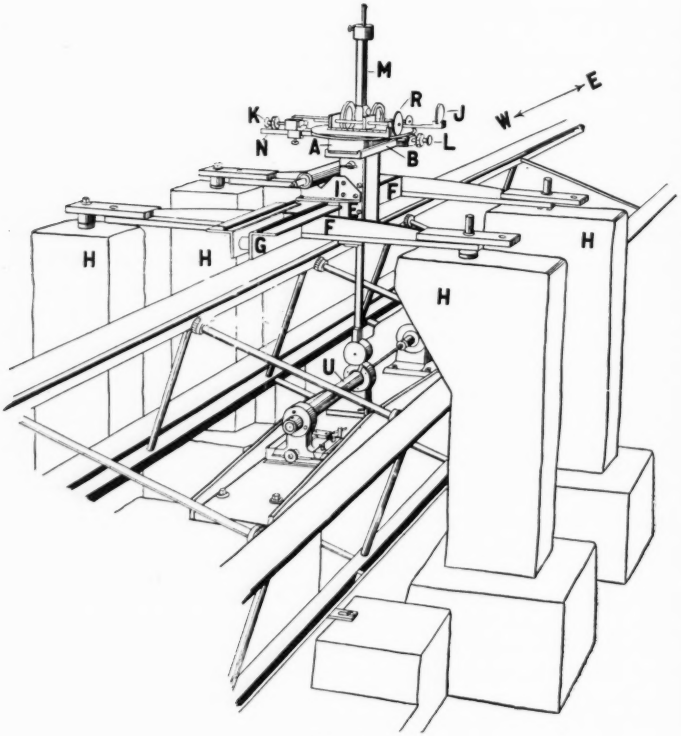

FIG. 3.

7. **The magnetometer supports.** The magnetometer supports, of brass, bronze, and concrete, as arranged in much of the work, are illustrated in Figures 3 and 4. The immediate support, *A*, was milled beneath to fit either of two slides *B*, *G*, which permitted motion east and west, parallel to the axis of the rotor. Slide *B*, by means of a projection from beneath, was movable vertically over a range of a

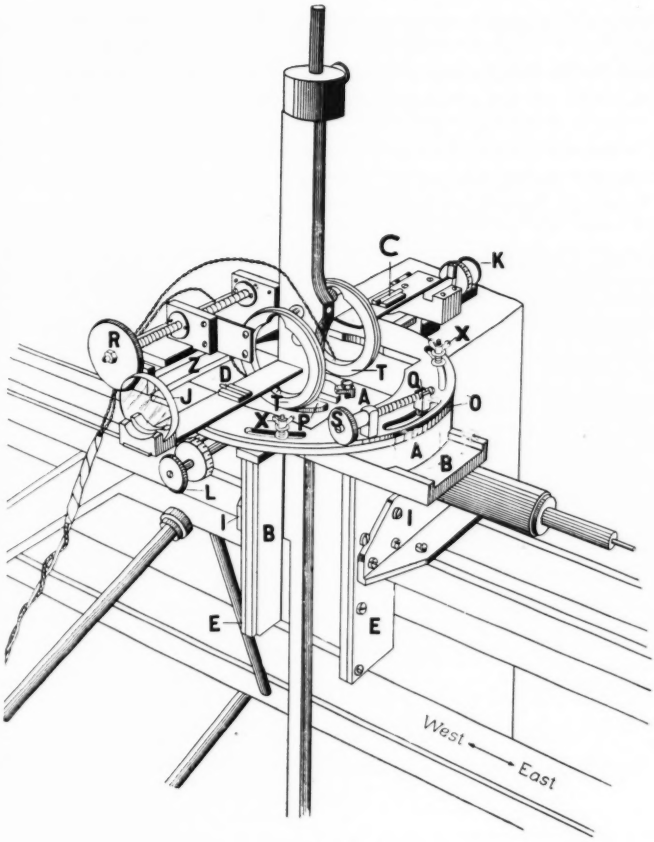

FIG. 4.

number of centimeters in the slide E. Clamps were provided for all the slides to prevent accidental displacement.

The slide E was screwed to angle pieces I, which were screwed to the H-form casting F, used in the earlier work; and the slide G was screwed and soldered to the same casting.

When A was mounted in the slide B, in its lowest position, the

center of the lower magnet was very nearly 10 cm. above the axis of the rotor and 5.8 cm. south of the vertical plane through the axis. When A was mounted in slide G, the center of the lower magnet was very nearly on the geometrical axis of the rotor. In the second case, the magnetometer was said to have an *axial* position; in the first case, provided the vertical plane ϕ through the center of the rotor and normal to its axis passed very nearly through the centers of the magnets, it was said to have an *equatorial* position.

In most of the early work, strong horizontal boards were screwed between the north and south projecting arms of the casting F in order to reduce vibration.

The casting F was firmly supported by four concrete piers H.

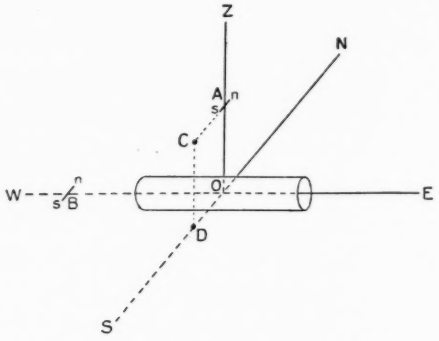

FIG. 5.

For the latter part of the work the angle pieces I were removed and the slide E was moved north about 5.8 cm. and was strongly secured by heavy angle pieces to the casting F in such a way that the center of the suspension lay in the vertical plane containing the rotor's axis. When, with this arrangement, the support A occupied such a position that the center of the suspension lay also in the plane ϕ, the magnetometer was said to occupy a *vertical-equatorial* position.

Figure 5 shows diagrammatically relative positions of rotor and lower magnetometer magnet in an axial position (B), and in equatorial positions (A, C, D), A being a vertical equatorial position and D (not used) being a horizontal equatorial position. The plane ϕ is the plane ACD. In the principal magnetometer positions which we have used the approximate distances were as follows: (1) Lower equatorial

(*LEQ*), $AC = 5.8$, $AO = 11.3$; higher equatorial (*HEQ*), $AC = 5.8$, $AO = 16.0$; lowest vertical equatorial (*LEQV*), $AC = 0$, $AO = 11.4$; standard vertical equatorial (*EQV*), $AC = 0$, $AO = 12.9$; near axial (*AN*), $OB = 12$; far axial (*AF*), $OB = 17$.

For the final work, in order to reduce possible errors from vibration, the whole framework *F* was covered with concrete, and the piers *H* were reinforced with concrete. The appearance of the magnetometer and its supports in this case is shown in Figure 6.

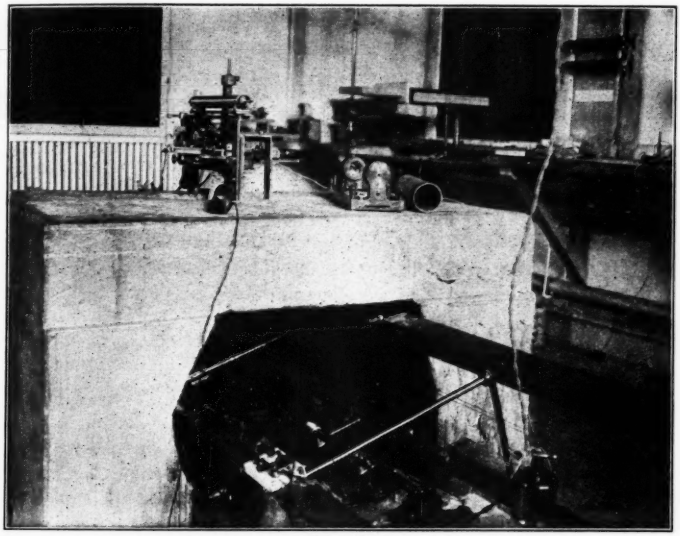

FIG. 6.

8. **The magnetometer controls.** In place of a single control magnet, as used in the earlier work, two minute magnets, *C* and *D*, Figure 4, have been used, both symmetrically arranged near the upper system. Motion of one of the magnets, *C*, with axis parallel to that of the system, alters the sensibility without altering the zero (with perfect adjustment); motion of the other, *D*, with axis normal to that of the system, alters the zero without altering the sensitivity (with perfect adjustment). Each magnet is provided with a slide for rough adjustment and a micrometer screw (*K*, *L*) for fine adjustment.

The adoption of this control system, which we have used for about seven years, has greatly lessened our labors in this and other work. Indeed the work described here would hardly have been possible without it. This system of control, in spite of its simplicity, is new, so far as we have been able to learn.

For this investigation it was necessary that the moments of the magnets be small to avoid the possibility of very objectionable eddy-current effects in the rotor. They were made from short pieces of hardened tungsten steel wire and had very small moments. The moment of the zero control changed from about 1.0 to about 0.7 c.g.s. unit, and that of the sensibility control from about 0.5 to about 0.4 c.g.s. unit from August, 1920 to December, 1923.

9. **The magnetometer sensibility coils.** In order to make it possible to measure the sensibility of the magnetometer at any time, two arrangements have been used. In one, a small pair of Helmholtz coils, easily removable, was attached to the lower end of the magnetometer case with its center approximately coincident with that of the lower magnet system and its axis normal to the axis of the system. Each coil, in the case of one magnetometer, had four turns; the radius was about 3.33 cm., and the mean distance between corresponding turns was 3.5 cm., so that the constant of the coils was about 1.05 gauss per ampere.

In the other arrangement, the two coils were mounted on the magnetometer support *A*, Figures 3 and 4, and their axes passed normally through the upper magnet on which the coil system was centered. These coils were intended to be, and were, except at first, placed so far apart that magnetometers could be exchanged without interfering with them. This arrangement has some advantages over the other, but has the disadvantage that the coils and the rotor act chiefly on different magnets, necessitating greater care in calibration. The constants of the various coil systems used did not differ greatly from one another.

10. **The main electrical compensating system.** To neutralize the earth's magnetic field in the early part of the work we used the continuously wound Columbus compensating frame. The wires on the frame were restretched, extra distance pieces were inserted, end coils were added to increase the breadth of the nearly uniform part of the field, and to make it more nearly symmetrical about the center, and the frame was mounted with precision with its longest sides normal to the magnetic meridian and its axis parallel to the earth's intensity.

This coil produced a field which was uniform along the axis of the magnetic part of the rotors within less than 1 part in 1200, and uniform over the central circular section of the largest rotor within about 1 part in 1000. We came to distrust this coil, however, partly because its dip and compensating current underwent changes which we could explain only, if at all, by assuming that warping had occurred; and we finally abandoned the coil for another of larger dimensions and better design.[25]

The new coil was designed to produce a uniform cylindrical field on the principle on which the Helmholtz double coil produces a uniform spherical field. A system of two infinite parallel wires distant a apart and traversed by the same current in opposite directions, together with an exactly similar system symmetrically placed and distant d from the first, produces a field the central portion of which is very nearly uniform, provided that $\frac{a}{d} = \sqrt{3}$.[26] If $d = 30$ cm. and $a = 52$ cm. this relation is very nearly satisfied, and calculation shows that the intensities at points distant 4 cm. from the central line in the direction of the field, and normal to this direction and to the central line, differ only by about 1 part in 10,000 and 1 part in 3000, respectively, from that at the central line.

For the coils wound on the new compensating frame a was 52 cm. and the mean value of d was 30 cm., the width (parallel to d) of each coil being about 1.7 cm. The length of the coil was 702 cm.

The construction of the frame is illustrated in Figures 3 and 6–10. There are four beams A of three-ply hardwood, 2.6 cm. square, running the full length of the frame. These beams were originally held in position by twelve sets of four thick-walled brass tubes B and C, all the tubes in each group having accurately the same length. A few of the tubes had to be removed later. The sets of tubes were nearly uniformly spaced along the frame. The beams were counterbored everywhere to the same depth to fit the tubes, which were threaded at the ends and fastened in place by brass screws.

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[25] Experience has convinced us that it would be much better to use, in place of a single coil system, two coils, traversed by different currents, one to compensate the horizontal intensity and the other to compensate the vertical intensity; also, that the wires should be stretched, without intermediate supports, between terminal machined plates. Suitable tension can be provided either by using sufficiently elastic wires or by using springs. In our last experiments on eddy currents we have used a frame built somewhat in this way for other experiments. We should prefer uninsulated wires, one or two diameters apart.

[26] See J. J. Thomson's *Electricity and Magnetism* (5th edition), p. 257.

The frame was fastened to low piers *D* on the floor with its length as nearly normal to the magnetic meridian as practicable and the brass tubes as nearly parallel and perpendicular to the earth's intensity as practicable. It was secured to brass lugs in the piers by brass angle pieces about a meter apart and brass screws, provision being made for both horizontal and vertical adjustment. An accurately machined piece of pyralin *F* was then fastened to each end of one beam and a single wire was stretched tightly along the beam. This wire was made level and the frame adjusted until the wire was normal to the meridian. With this wire as a guide, pyralin blocks, *F*, machined to hold an exact number of wires and to bring them accurately into the required positions, were placed at intervals along each beam and permanently fastened with brass screws. These pyralin blocks were intended to be placed opposite the ends of the tubes *BB*; but we discovered too late that this precaution had not been taken, and some of the blocks were not even close to the tubes.

The upper and lower coils were at first wound continuously with enameled wire, and later with silk-covered wire; but the insulation, injured in the winding, broke down and introduced uncertainties as to the exact compensation of the earth's field.

Each coil was therefore finally replaced by two coils, each consisting of nine turns of No. 20 d.s.c. wire, the strands of one coil being alternate with those of the other; and such terminal arrangements were made that by means of three double throw switches and a voltmeter the voltages over the alternate strands, as well as the insulation resistance between them, could be readily measured. When the arrangements were completed, the resistance was always far higher than necessary.

Sagging of the wires between the pyralin blocks, against which they were kept pressed, occurred twice after the coils were wound, but was done away with by means of a stretching device, illustrated in Figure 10, attached to each corner of the frame at the west end. This device did not act continuously and permanently, however, until stiff springs *SS* were inserted, and kept under strong pressure, between the frame *A* and the nut *B*, after which there was no more trouble.

To keep the cross-section of the coil system rectangular, four diagonal braces of brass, one of them with a ring at the center (Figure 7) to admit the driving shaft, were originally inserted. One brace near the end was later removed, to extend the shaft through the frame, and another was modified as in Figure 7. Later still, when the heavy concrete was cast about the magnetometer, four adjustable braces, acting between the upper part of the frame and the concrete piers *H*,

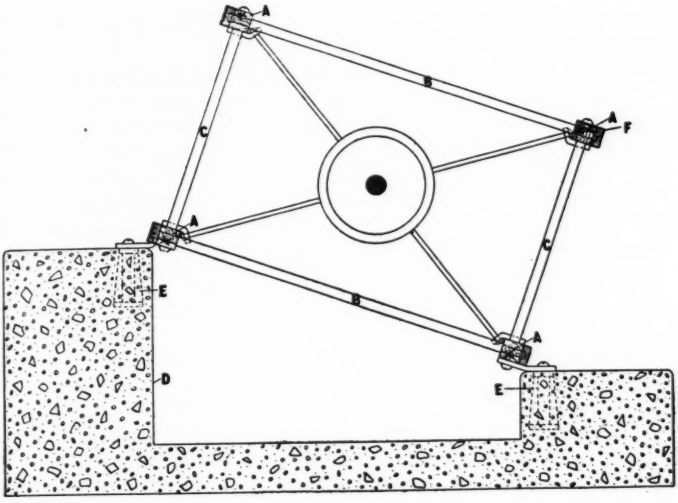

FIG. 7.

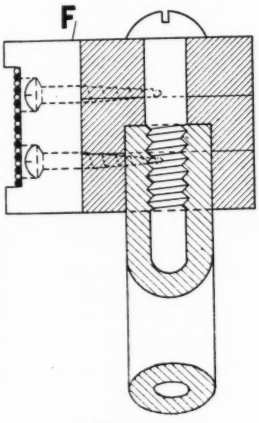

FIG. 8.

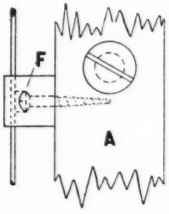

FIG. 9.

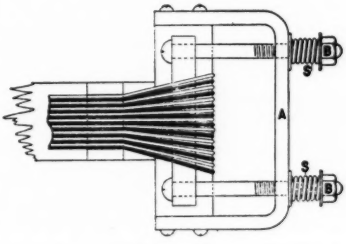

FIG. 10.

were added to fix more certainly the position of the central part of the coil system.

Several years ago the axial component of the intensity produced by the frame in a central horizontal cylindrical space about 2 m. long and 10 cm. in diameter was measured with an induction balance and was found uniform to 1 part in 2500. Later on, in August, 1923, the axial intensity (approximately parallel to the earth's intensity) along a central horizontal line about 40 cm. long was found uniform to about 1 part in 4000. The horizontal intensity over the same length was found to vary about 1 part in 400, the frame having become distorted. By means of the braces the upper part of the frame was therefore readjusted until the upper coil, whose west end we found to have moved a few millimeters south, was parallel throughout its length to the lower; and gages with fixed points of reference were set along the frame so that any future displacements could be accurately read. Many readings by means of these gages during the remainder of the work showed that no displacements beyond a fraction, usually a small fraction, of a millimeter occurred. Induction balance tests just after the frame was readjusted and at two later dates, one of them near the end of the work, showed that the horizontal component over the central line referred to was uniform to about 1 part in 14,000, 1 part in 7,000 and 1 part in 14,000 or better.

11. **The subsidiary electrical compensating system.** Changes in the elements of the earth's magnetic field, secular, diurnal, etc., together with originally imperfect construction and adjustment and later possible alterations in the compensating frame itself, and the necessity of testing for the effects of small variations from correct compensation, made it necessary to install a subsidiary compensating system. If we suppose that the *mean vertical* intensity acting upon the rotor is annulled by the current in the main compensating coil, there will, in general, be left acting on the rotor: (1) a small axial intensity, which may or may not be uniform; (2) a small uniform horizontal intensity H_0 (Fig. 11); (3) a small nonuniform north and south horizontal intensity h (Fig. 11) which may be considered (except so far as very close sources are involved) proportional to the axial distance from the center of the rotor, and thus opposite on opposite sides of the center; and (4) a small nonuniform vertical intensity, v, proportional, like h, to the axial distance from the center of the rotor.

We at first used no subsidiary system, and we made several changes before adopting the final arrangement, which alone will be described here.

To compensate the residual intensities (1), (2), and (3) we used three pairs *A*, *B*, *C* of circular cylindrical coils of enameled copper wire carefully wound on bakelite bobbins. The coils of pair *A*, used to compensate (1), were about 6 cm. thick, of No. 16 wire; the others about $7\frac{1}{2}$ cm. thick, of No. 14 wire. All had internal and external diameters of about 12 cm. and 22 cm. The coils of each pair had practically equal constants and were connected in series. All the coils were mounted with their centers in the equatorial plane of the rotor, one coil of each pair being as far north of the axis as the other was south.

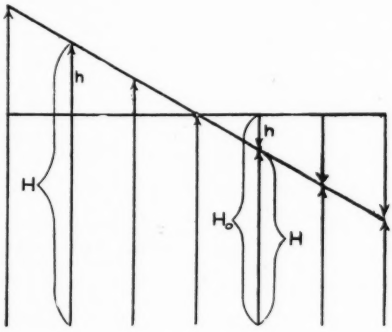

FIG. 11.

Coils *A* were mounted with their axes parallel to that of the rotor, very nearly on a straight line passing through its center, and about 433 cm. north and south of its axis. They were traversed by currents in the same direction and produced a very nearly uniform axial field throughout the rotor.

Coils *B* were mounted 400 cm. north and south of the rotor's axis, with their axes in its equatorial plane, and (finally) in the horizontal plane passing halfway between the magnets of the magnetometer when in the equatorial position used in the last experiments (earlier they were mounted lower). They were traversed by currents in the same direction and produced a field which was very nearly uniform over the magnet system, the rotor, and the compensator.

Coils *C* were mounted about 416 cm. from the vertical plane through the rotor's axis, with their axes parallel to this axis and in the same horizontal plane which contained the axes of *B*. They were traversed

by currents in *opposite* directions, when used normally, to produce a nonuniform horizontal intensity at the rotor, normal to it, and proportional to the axial distance from its center; but the currents could be given the *same* direction for purposes of magnetometer testing (see below, § 27).

The south coils of the pairs B and C, and the uppermost part of the south coil of the pair A, are visible in the lower left part of Figure 12, which shows much of the experimental apparatus in its final state.

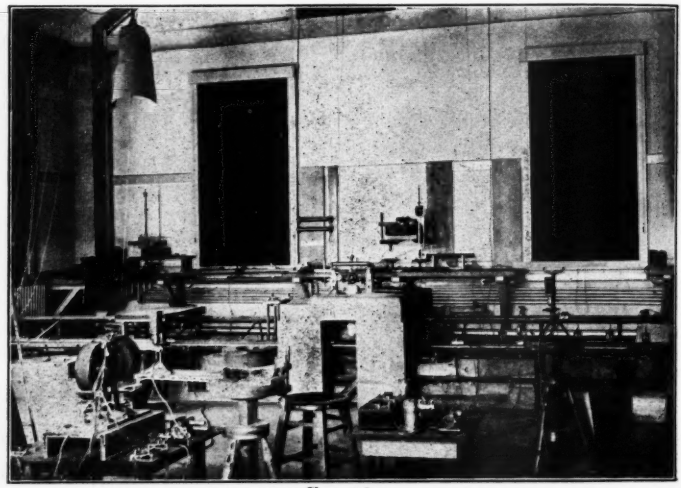

FIG. 12.

The three coils north of the magnetometer were mounted over a brass plate on a concrete block in a partially double-walled box outside the laboratory, and were connected to the interior by lead-covered twisted wires. One coil of each pair was mounted on a tripod with adjustments for altitude and azimuth, which, however, proved unnecessary.

The field of each of the coils in the region occupied by the rotor was, of course, not strictly uniform. The maximum variation for any coil, however, was less than 1% of its value at the center of the rotor. Hence, especially as that central value was itself never greater than about 1% of the earth's horizontal intensity, the degree of uniformity

of the field produced by the combined compensating system in the region occupied by the rotor was very high.

The horizontal intensity, normal to the rotor and directed south, produced by system *C* along the east half of the rotor, when the axis of the inner coil was directed west and that of the outer coil east, for a current of 1 ampere in the coils, is given in Figure 13. For the other half of the rotor the intensity was equal and opposite. When the

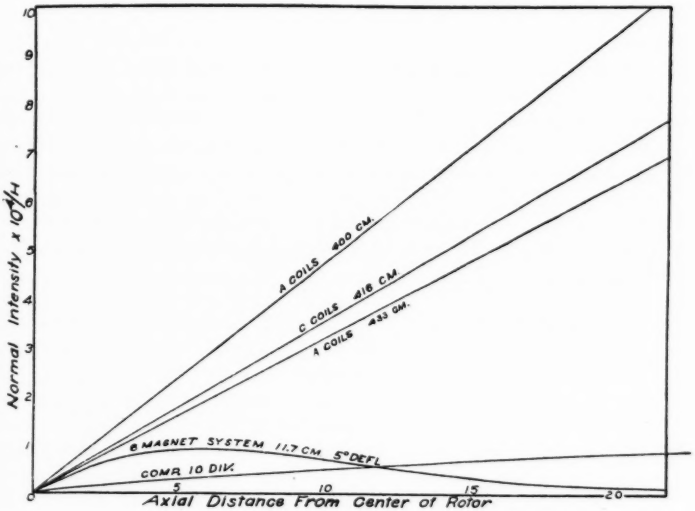

FIG. 13.

current in the outer coil was reversed, the (axial) intensity acting on the rotor was about 0.0044 *H*, where *H* denotes the horizontal component of the earth's intensity.

Currents of 1 ampere in coil systems *A* and *B* produced at the rotor intensities about 0.0063 *H* and 0.011 *H*.

In order to annul any nonuniformity in the magnetic field acting on the rotor in the direction of the axis of the main compensating frame, an extra coil or loop consisting at first of one turn and later of two turns, parallel and near together, was provided.

In the east-west direction the wires of this loop were of the same length as those of the frame. The median plane of the loop coincided

with that of the frame, and its axis, when its wires were parallel, coincided with that of the frame. By means of two slides at the west end of the loop, and two similar slides at the east end, each with a range of 4 cm., the west ends of the wires could be moved apart and the east ends closer together, or vice versa, while the distance between them at the center remained constant (60 cm.).

When the slides of the double loop are each displaced, outward at one end and inward at the other, the maximum amount 4 cm., and the current flows through all the wires in the same direction, calculation shows that over the central 40 cm. containing the rotor the variation of the total intensity thus produced is about 0.0004 of its value at the center. With this arrangement the double deflections produced by rotating copper with the field through the rotor parallel to the axis of the frame annulled could be made zero. But under these conditions the deflection produced by rotating permalloy and annealed steel rotors did not differ from those for the same mean compensating field when the wires were all parallel. The use of the loop was soon abandoned in favor of the extra coil system whose final arrangement has already been described.

12. **The variometers.** In order to be certain of the limits within which the earth's magnetic field in the region occupied by the rotor was compensated, it became necessary either to make very frequent observations with inductors, a course consuming more time and energy than were available in the best hours of the night, when the rotation work had to be done, or else to use three variometers, whose readings could be obtained quickly at any time and could be used in connection with less frequent inductor observations. This latter course was followed in all of the latest work, and in some of that which preceded.

Horizontal intensity and vertical intensity variometers and a declination variometer were installed in the laboratory. The first was a Wild-Edelmann instrument with a bifilar suspension; the second a Lloyd balance of the Wild type by Edelmann; the third a C. I. W. magnetometer (No. 4). As the instruments were finally arranged, 1 mm. deflection was produced on the first by a change of about 1 part in 2244 in the horizontal intensity; 1 mm. on the second by a change of about 1 part in 3400 in the vertical intensity; and one scale division on the third by a change of 1.5 minutes in the declination. The Lloyd balance, as is usually the case, could not be relied on over great lengths of time, the base line being subject to unavoidable changes; but it could be used over short intervals with advantage.

The declination instrument was quite reliable, the horizontal intensity variometer usually so.

The readings of all three instruments were, of course, affected by the currents in the compensating coils, but these were readily allowed for. Most of the observations, indeed, were made when these currents were zero; or when all were zero except that of the main compensating frame, whose two coil systems, exceedingly close together, were then connected in opposition. They were sometimes connected in this way in order to let the current have more time to become steady while the variometers were being read.

The temperature coefficients of the horizontal and vertical variometers were obtained by means of values of the intensities furnished by the Coast and Geodetic Survey from observations at the Cheltenham Observatory. These coefficients for the horizontal and vertical intensity instruments, as used in the more recent work, were -2.5 mm. per degree C. and -1.05 mm. per degree C., respectively.

Inasmuch as the observations show that the compensating frame itself underwent small changes with the time, the use of variometers was not so reliable as frequent tests with inductors; but the observations show also that the small changes occurring were of little importance when the magnetometer adjustments were approximately correct.

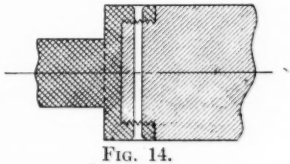

FIG. 14.

13. **The types of rotors.** After some magnetic tests made for us by Mr. D. M. Wise with iron cylinders of different diameters and lengths, it was decided to adhere to rotors of about the dimensions used in the Columbus magnetometer experiments. Indeed, many of the rotors we have used are Columbus rotors, usually modified.

The type of rotor, with half-inch journals, used in Columbus, was also used in the earliest part of the more recent work. The terminal parts, including the journals, were of bell-metal, and were soldered to the magnetic material. Two of the rotors, of Heusler's alloy, were both originally of this type, except that the journals were cast integral with the main cylinder, and the whole ground into final form. One of these, known as Heusler's alloy II, was left in the original form; the other, Heusler's alloy I, was later modified as shown in Figure 14, which gives a longitudinal central section of one end. The end of the main cylinder was threaded and a brass cap integral with a half-inch bell-

metal journal was tightly screwed onto it and pinned in position. At the other end of the bell-metal journal, and shrunk into it, was an eighth-inch journal of Stellite II, as in Figure 15. All the journals were accurately coaxial cylinders, the corresponding journals at the two ends of each rotor had very nearly equal diameters, and the stellites were carefully lapped and polished.

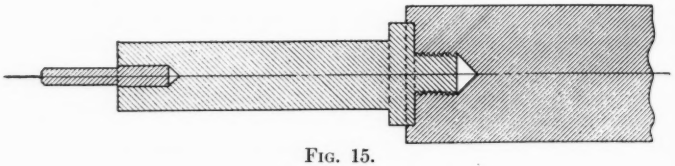

FIG. 15.

Four other styles of magnetic rotor were used, improvements being made as the work progressed. The first was similar to that of the old rotors, but with stellite journals added as in the case of Heusler alloy I. The second is illustrated in Figure 15. The bell-metal journals were screwed as well as soldered (with certain exceptions, as permalloy) or pinned, to the magnetic material; and the stellite journals were added as in the other rotors.

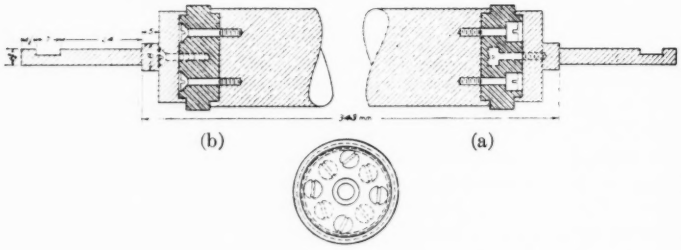

FIG. 16.

In the third type, illustrated in Figure 16 (a), each journal piece was cut out of one piece of drawn phosphor bronze. A disc of bakelite dilecto about 1 cm. thick was screwed to the flange, and the disc then also screwed to the rotor, both sets of screws being deeply countersunk in the bakelite. In the fourth, Figure 16 (b) and (c), the construction was somewhat similar, but mechanically much stronger. The bakelite

disc was screwed to the rotor, then the bronze flange was screwed to the disc, both sets of screws passing through nearly the whole thickness of the bakelite.

The journals of these two types of rotors were carefully worked to true and nearly equal cylinders and to fit with minimum practicable play the three-sixteenth inch agate bearings described below. The lengths of these rotors, from journal proper to journal proper, were made almost precisely the same, 343 mm., and the two ends were almost exactly alike, including the location of the notches. The approximate dimensions of the journals are given in millimeters in Figure 16 (a) and (b). All parts of the figure are drawn to the same scale.

14. **The magnetic parts of the rotors.** Steel I and Steel III were cut from the same rod of commercial cold-rolled steel, which was not analyzed; Steel II and Steel IV were similarly cut from a smaller rod of the same material. Steel II and Steel IV were annealed, the other two were not, except as some annealing occurred while the journals were being attached.

Cobalt I and II were machined from a single rod cast for us by the Deloro Mining and Reduction Company, through the kindness of Mr. S. B. Wright. The material from which the Cobalt II was made we annealed, but the process had no appreciable effect on its properties. The complete rod was probably annealed in Deloro. No special analysis of the material was made, but from analysis of other specimens from the same company it probably contained about 95 or 96% cobalt, 3% nickel, 1% iron and smaller quantities of carbon, silicon and sulphur. Cobalt, as is well known, is difficult to cast and to work and our rotors were not homogeneous. Cobalt II was better than Cobalt I, which was rotated in Ohio, and was statically balanced.

The Armco Iron rotor and compensator were cut from a single rod, for which we are indebted to the Page Steel and Wire Company.

The Norway Iron rotor and compensator were cut from a single rod of the commercial material.

Electrolytic Iron I and its compensator were cast from the same heat at the Bureau of Standards, and contained only minute quantities of impurities, carbon (0.03%) being much the most prominent. The rotor was far from homogeneous, but was statically balanced, though only after the chief experiments with it were made. For this rotor and compensator, as well as for several described below, we are indebted to Dr. Burgess, Mr. Jordan and Mr. Neville.

Electrolytic Iron II was forged by the Westinghouse Electric & Mfg. Company from an ingot of Yensen's pure electrolytic iron, melted in an induction vacuum furnace. We are indebted to the company and Mr. Yensen for the material. The rotor is apparently very nearly homogeneous and only a slight improvement was made by static balancing.

The nickel rotors were all machined from Boker's soft, pure nickel.

For the Iron-Cobalt, Cobalt-Nickel, and Iron-Nickel (Hopkinson's alloy) rotors and compensators, we are indebted chiefly to Dr. Burgess and Messrs. Jordan and Neville of the Bureau of Standards. Melting was done, as was the case with Electrolytic Iron I and its compensator, in crucibles of zirconium silicate, or lined with zirconium silicate, in an induction furnace.

The Iron-Cobalt (Preuss's alloy) rods contained about 34% cobalt, 0.2 and 1.5% silicon, 0.04 and 0.06% carbon.

The Cobalt-Nickel (Bloch's alloy) rods contained about 45% nickel, 54% cobalt, nearly 0.9% silicon and 0.3% carbon, and minute quantities of manganese, phosphorus and sulphur.

The Iron-Nickel (Hopkinson's alloy) rods contained about 24.5% nickel, 75.0% iron, and small quantities of silicon, etc. After being cast these rods were cooled for some time to $-15°$ C. (rotor) and $-18°$ C. (compensator), through the kindness of Dr. C. W. Kanolt.

All these rods were cooled very slowly in their moulds in the furnaces after casting, and thus came out annealed. All the rotors required balancing, the Preuss's alloy least, Hopkinson's alloy most.

For the permalloy[27] rotor and compensator we are indebted to Dr. Arnold and the Western Electric Company who were so kind as to supply us with the most nearly homogeneous rods they could produce and at a time when the development of the material was still in progress. The rods were "dead soft annealed," and the rotor required little (static) balancing, although its density was about 1% less than it would have been if it could have been thoroughly forged.

Permalloy, as is now known, contains about 80% nickel and 20% iron, and is extremely susceptible in weak fields. We could not, of course, take full advantage of this susceptibility on account of the considerable demagnetizing factor entailed by the shape of our rotors.

Heusler's alloy I and II were cast in an extremely cold room from "extra pure" aluminum, pure manganese and electrolytic copper, by

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[27] H. D. Arnold and G. W. Elmen, *Journal of the Franklin Institute*, **195**, 621, 1923.

Professor W. J. McCaughey of the Ohio State University, and one of us. The ingredients were inserted in atomic proportions, melted in a graphite crucible, and cast in a heavy, specially prepared, graphite mould, with projections for journals greater than half an inch in diameter. The two rods were cast from separate heats, and had considerably different moments in weak fields. They were cast with axes vertical and are somewhat porous at the upper ends. Heusler's alloy II was not balanced; Heusler's alloy I was statically balanced for the final work.

TABLE I.

THE ROTORS AND SOME OF THEIR CONSTANTS.

1	2	3	4	5	6
Rotor	Diameter	Weight	Moment*	K (end correction)	Conductance†
	cm.	kg.			
Steel I (cold-rolled)	3.10	1.88	0.76	2.1%	0.122
Steel II "	2.31	—	0.62	3.2	——
Steel III (annealed)	3.09	1.86	1.00	0.8 (1.5 AF)	0.122
Steel IV "	2.32	1.07	0.85	1.0	0.064
Electrolytic iron I	3.18	1.95	1.03	0.6	0.170
Electrolytic iron II	3.32	2.15	1.00	0.8	0.197
Norway iron I	2.79	1.52	0.79	1.8	0.127
Armco iron	2.56	1.27	0.71	1.9	0.110
Cobalt I	3.20±	2.21	0.82	1.7	0.052
Cobalt II	3.08	2.02	0.82	1.9 (2.0 AF)	0.051
Nickel I	2.26	1.14	0.75	1.7	0.066
Nickel II	3.04	2.07	0.94	0.9	0.157
Nickel III	3.01	2.01	0.93	1.0	0.153
Preuss's alloy (Fe-Co)	3.28	2.08	1.09	0.6 (0.4 AF)	0.148
Bloch's alloy (Co-Ni)	3.20	2.04	0.70	2.5	0.044
Hopkinson's alloy (Fe-Ni)	3.25	2.08	0.44	5.1	0.021
Permalloy (Ni-Fe)	3.34	2.32	1.25	0.0 (−0.25 AF)	0.130
Heusler's alloy I	3.14	1.57	0.48	4.6	0.043
Heusler's alloy II	3.13	1.60	0.37	8.1	0.045
Copper	3.07	2.07	—	—	——
Wood	3.6	1.27	—	—	——

* Relative to Steel III (approximate).
† Relative to copper rotor.

The rotors made from drawn or forged rods of nickel, iron and steel all appeared to be very nearly homogeneous, and little or no improvement could be made in attempts to balance some of them statically.

The Vibration Specialty Co. of Philadelphia endeavored to balance a number of the rotors both dynamically and statically. They were able in some cases to improve slightly on the static balancing done in our own laboratory, but their machines were not sufficiently delicate to balance our rotors dynamically. The balancing was done on the advice of Mr. Gano Dunn.

The diameters of the rotors in their final states, their weights, and some of their other constants are given in Table I.

15. **The bearings, countershafts, bedplates, etc.** The bearing piece in which the rotor moved was a great improvement over that used in the earlier investigation. It is illustrated in Figure 17. The

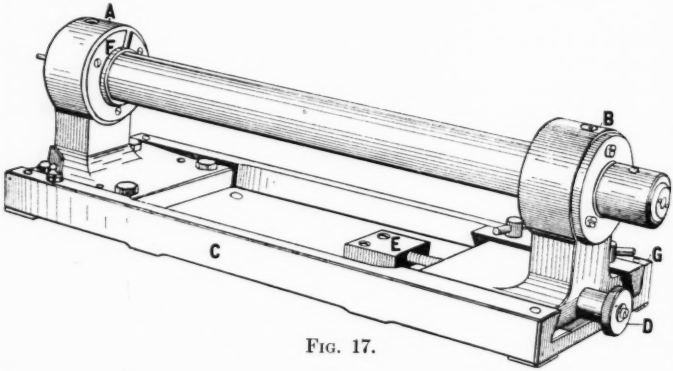

FIG. 17.

bearing block *A*, at the east end, was bolted to the base *C*. At the west end the base was provided with an accurately made slide and jib *G* in which the heavy block *B* could be moved longitudinally either directly or by the screw *D*. It could be clamped in any position by the jib and its screws. Eight projections (four at first) on the bottom of the base were planed for attachment to a heavy bedplate, Figure 3, by thumb screws, some of which are shown in Figure 3. The base was provided with pins (removable in the latest part of the work) which fitted precisely into holes in the bedplate, making it easy to place it always in the same position.

In some of the earliest work the original half-inch lumen metal

rotor bearings were used. Later, new lumen bearings, made with greater precision, were substituted for them, as were also, sometimes, half-inch babbit bearings in bronze castings shaped externally like the original lumen bearings, so as to fit the blocks *A* and *B*. Half-inch bearings of lignum vitae, bakelite, and San Domingo mahogany were also used on some occasions.

In much of the work a half-inch bearing was used at the driven end of the rotor, and a small bearing of bronze, or later, in most cases, agate, fitting the eighth-inch stellite journals, was used at the other

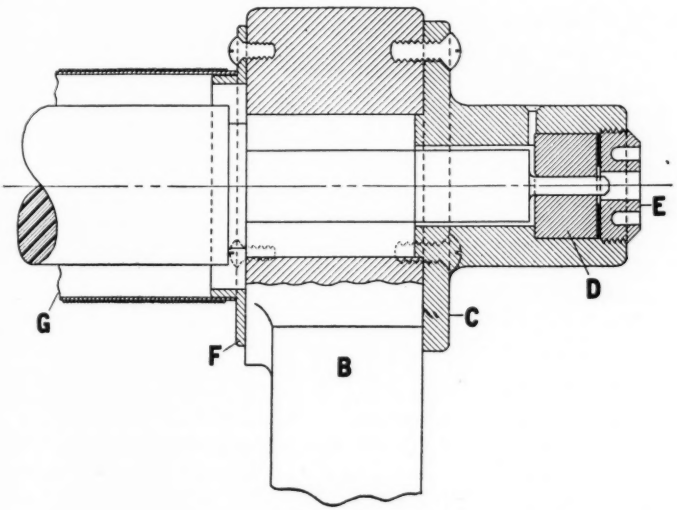

FIG. 18.

end. The construction is shown in Figure 18, where *D* is the bronze or agate. A lock nut, which is not shown in the figure, was later provided, as on one occasion the agate became loose.

For the last series of experiments, beginning in August, 1923, three-sixteenths inch bakelite bearings, and, a little later, three-sixteenths inch agate bearings, were provided for both bearing blocks, the principles of construction and mounting being similar to that of Figure 18, with the details different, as indicated in Figure 19. The bearings had to be moved inward, as the journals were closer together, and the blocks

had to be bored out to admit machined castings heavy enough to hold the agates or bakelites with sufficient firmness.

For the excellent work done in making the agate bearings we are indebted chiefly to Mr. A. F. Erickson, whose services were secured for us through the kindness of Mr. Olof Ohlson of the Waltham Watch Company.

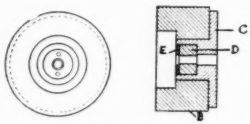

FIG. 19.

The bearing piece was for a long time mounted on the bronze bedplate, Figure 3, used in Columbus. In the later work this was replaced by a considerably heavier bedplate, Figure 6. This plate was much freer from magnetic material than the older plate; instead of the sloping ribs it was provided with machined horizontal ribs, which were exceedingly useful in several ways; and better workmanship on its mounting surfaces made more precise adjustments practicable.

At the east end of the plate was mounted the small countershaft used in Columbus (including, later, a new rod freer from iron), Figures 3 and 6. In all the later work each of the bearing blocks for this countershaft was electrically insulated from the bedplate.

When the rotor was driven by a half-inch journal at the east end, the countershaft was connected with the journal by a small rod (finally of phosphor bronze and one-fourth inch in diameter) and a chuck (also finally of phosphor bronze) arranged as in Figure 20. The

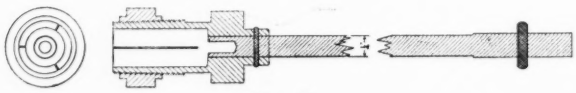

FIG. 20.

rod, in the later work, passed through two guides of lignum vitae, Figure 6, which served to protect it from accident and which were originally intended as bearings to prevent vibrations — an arrangement which proved impracticable.

The rod and chuck, carefully balanced to prevent vibration, used in the last series of observations, on rotors with three-sixteenths inch journals, are illustrated in Figure 21.

The small countershaft on the bedplate was connected by a somewhat thicker shaft, five-sixteenths inch in diameter, about 42 cm. long, and provided with suitable chucks, to a third and larger shaft. In the earlier part of the work the third shaft was provided with a pulley and driven by a belt, or by a gear box, connected with the source of power. In the later work it was directly connected by flexible couplings (steam hose) and one or more additional shafts, connected by means of steam hose, to the gear box. In all of the work of the last two years, there were six of these larger shafts between the small shafts and the gear box.

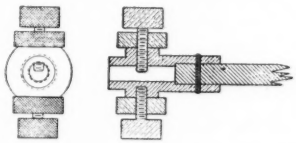

FIG. 21.

The bedplate and countershaft bearing blocks were bolted to brass cones or rods embedded in concrete piers cast on the concrete floor, with the axis of the rotor bearings (approximately) level and parallel with the wires of the compensating frame; and with the center of the rotor close to the center of the frame.

The later (and heavier) bedplate was bolted to the piers directly at twelve points; and at four points, under the central portion, to heavy brass rods ending below in concrete.

16. **The source of power.** The ideal source of power for this investigation would have been compressed air; but suitable compressed air appliances for producing and controlling with precision the speeds of our rotor were not available. The next best source of power is a polyphase electric system, but this also was not available when our work in Washington began. Hence we had again recourse to single-phase alternating current motors.

A large repulsion motor of this kind was used in the early part of our work in 1914, but it was abandoned for compressed air, because we suspected it of introducing a systematic error. Of this, however, there was no experimental evidence; and we used a small motor with repulsion start in the work of 1916–17, with an error which was undoubtedly small. As a possible source of systematic error with repulsion motors we had suspected the commutator, whose function is

rectification; and Mr. Gano Dunn, who was not aware of our suspicion, later suggested the same idea to us and informed us that it was correct. The only commutator motor which we had then tested (in its proper position and by means of the astatic magnetometer) had shown no certain systematic effect; but larger motors of the same type, by which it had to be replaced to get sufficient power for the higher speeds, all showed distinct effects on reversal of direction of rotation.[28] One of these larger motors was mounted inside the building just inside the east end of the frame; the others which were tested, in a concrete pit east of the building and connecting with the interior through a small tunnel. The motor, when mounted in the pit, was about 6.7 m. from the magnetometer. Two of these motors were used in the rotation experiments, and the effects referred to were determined and allowed for; but we thought it desirable to eliminate them entirely. Hence we concluded to use a three-phase system when that became practicable, and finally mounted in the pit a General Electric 5 h.p. three-phase slip-ring motor, which was used in all the later work without appreciable systematic error.

17. **The speeds.** For this work it was important to obtain speeds as nearly uniform as practicable, and independent of the direction of rotation. Belts did not prove satisfactory, and we soon adopted spiral gears. Two boxes were made in the shop to hold bronze gears cut by the Philadelphia Gear Works, both with plain cylindrical bearings. Later, one box had thrust ball bearings inserted, and was substituted for the other, used first. One shaft of the box was joined to the motor shaft by a flexible coupling, the other to the countershaft most remote from the rotor. The speeds obtainable were 1/1, 2/1, 1/2, and 2/7 times the speed of the motor, which was a four-pole machine operating at about 30 r.p.s. at the frequency 60 cycles per second and with the secondaries short-circuited. The motor was started with a standard controller, and had also inserted in the secondaries three adjustable extra rheostats with resistances much greater than those of the controller. Thus the speed could be varied, between wide limits, either by considerable steps or by small gradations.

In much of the work a Weston magneto, repeatedly calibrated, was connected by a flexible coupling to the gear shaft operated directly by the motor, and speeds were determined from the voltmeter readings (and, in the later work, the magneto temperature). In nearly all of

[28] For the opportunity of testing various motors without their purchase, we are indebted to Mr. Eastman of the National Electric Supply Co.

the recent work, however, the magneto was used only as an adjunct, and the speeds directly determined with a standard chronometer and a standard chronograph. For the loan of this chronograph we are indebted to the Coast and Geodetic Survey. The substitution of chronograph for magneto considerably increased the labor involved, but until it was done we were often uncertain whether apparent speed troubles were real or were due to magneto imperfections.

The motor ordinarily required from 5 to 7 seconds to reach approximately full speed. It was operated ordinarily for 30^s (or sometimes 35^s). Speeds were determined from the time of 500 revolutions (approximately 16^s, except during special tests) of the motor near the end of that interval.

The average behavior of the motor as finally used when developing the maximum power, with the gear ratio 2—1, is illustrated in Table II, which gives a mean time of each hundred revolutions of the motor between successive chronograph breaks for the 700 revolutions ending with the last break before the magnetometer was read. Each mean is obtained from thirty-five runs from four sets of observations taken at random from the last series (Series B). Group 1 is the earliest in time, group 7, the latest.

TABLE II.

MEAN TIMES OF SUCCESSIVE HUNDREDS OF REVOLUTIONS OF MOTOR.
GEAR RATIO 2—1.

Order of group	1	2	3	4	5	6	7
Mean time in seconds	3.34_2	3.28_2	3.27_7	3.27_1	3.27_3	3.27_4	3.27_4

18. **The upper compensating system.** In order to make the sensibility of the magnetometer and its zero approximately independent of the current in the main compensating frame, it was necessary, as in the earlier experiments, to provide special coils, Figure 4, *T*, *T*, to compensate approximately the horizontal intensity of the earth's magnetic field in the region occupied by the upper magnet system. As in the earlier experiments, two coils forming an approximate Helmholtz pair, centered on the upper magnet, were used. In the final apparatus each coil consisted of a single turn of wire about 6.8 cm. in diameter, wound on a vertical ring of bakelite dilecto, the rings being screwed to brass bases movable for axial adjustment in a brass slide

by a right-and-left-handed screw R in such a way that the magnet was always approximately at the center of the system. As before, the coils were connected in series with the main compensating coil and in such a way as to oppose the earth's horizontal intensity.

The brass slide Z was screwed to a horizontal circle coaxial with the suspension and moving in a second coaxial circle O screwed to the magnetometer support A. A tangent screw S provided slow motion in azimuth.

By the axial motion of the coils the sensibility was easily made identical, or nearly identical, when the current was on or off; and by the motion in azimuth the zero was made identical, or nearly identical, without the necessity of adjusting the control magnets. These arrangements were very great improvements over those of the earlier experiments.

The control magnet slides, already referred to, were mounted on the fixed circle O.

19. **The method of calibration. Slow rotation during calibration.** The method of calibration used in nearly all the work was much superior to that used in the first magnetometer investigation, and involved three processes A, B and C, including the use of three coils as primary, secondary and reduction standards.

In process A it is necessary for convenience and precision that the rotor be in rotation, so that a mean effect is obtained; otherwise readings must be taken for several equally spaced position angles of the rotor, as was done in the earlier work. When the earth's field is just compensated this necessity arises chiefly from the fact that the field about the magnetometer magnets due to the residual magnetism of the rotor, in both direction and magnitude, is in general a function of the position angle. When the earth's field is not entirely compensated there is a similar effect on the magnetometer due to the fact that the rotor is not in general magnetically symmetrical about its axis.

In processes B and C, where only *ratios* of sensibilities are involved, slow rotation, when necessary, is required only on account of a rotor's lack of axial symmetry.

Even with unsymmetrical rotors no appreciable error is made in process C, and only minute errors in process B, if the rotor is kept at rest.

Throughout nearly all of the investigation described here the rotor, in processes A and B, was rotated slowly and uniformly either by a small and remote induction motor, or by the motor used in the princi-

pal experiments, the speed in this case being reduced either by gears or by suitable impedances in the circuit or circuits. In some cases, for special reasons, the calibrations were made at the speeds of the principal experiments.

20. **Process A. The magnetometer or absolute sensibility.** In process *A*, the sensibility of the magnetometer itself, largely independent of the particular rotor and compensator under investigation, was determined by reversing a minute known current *I* through the small *sensibility coils* (the *secondary standard*) centered on the upper or lower magnet, and adjusting the sensibility control magnet until the desired deflection, *A*, was obtained. This is the *magnetometer*, or *absolute*, sensibility.

In process *A*, as also in processes *B* and *C*, observations were made on a regular time schedule, reversals, alternately in opposite directions, being made at equal intervals. In process *A* the readings (and reversals) were usually 30^s apart.

This sensibility was ordinarily determined at the beginning and at the end of each series of observations. Large deflections, of the order of 20 cm., read to tenths of a millimeter, were ordinarily used in order to reduce the accidental error, with the result that quite sufficient precision was obtained, although the determinations were usually made just before the beginning and just after the end of the best part of the night. Usually the sensibility changed but little during a long series of observations, so that the mean of the two sets was applicable with negligible error to all of the series. In the last series of observations the average absolute error in the sensibility made in this way was about one-eighth per cent. Interpolation was used when necessary. For some (shorter) series the sensibility was obtained only at the beginning or at the end.

As several absolute sensibility coils were used in the course of this work, sensibilities obtained with other coils are all reduced to those which would have been obtained with the coil used exclusively in the last series of observations.

21. **Process B. The rotor sensibility and the ratio of rotor sensibility to magnetometer sensibility.** In process B the ratio of the *rotor sensibility* to the magnetometer sensibility was obtained. To form the *primary standard*, a brass tube, *G*, Figure 18, with length and internal diameter somewhat greater than those of the magnetic part of the largest rotor, was uniformly wound on a lathe with a single-layer solenoid of insulated copper wire. One end of this solenoid was

soldered to the adjacent end of the tube, and the other end of the solenoid and the adjacent end of the tube were soldered to two twisted leads. The tube was slipped over the rotor and then held centrally and coaxially in position by two properly fitting rings *FF* attached to the bearing blocks *A*, *B*, Figures 17, 18. By obtaining the deflection *B* which would be produced by the reversal through this solenoid of the same current *I* as that used in process *A* the *rotor sensibility B* could be determined.

If *A* denotes the deflection for a given current produced on reversal in the first case, and *B* that in the second case, the ratio $Q = B/A$ of the two sensibilities is fixed for a given rotor in slow rotation and a fixed position and condition of the magnetometer.

In general the magnetometer sensibility was determined, as already stated, immediately in connection with each group of rotation observations at night. The rotor sensibility in the latest work was not ordinarily determined with the magnetometer sensibility equal to its value in the principal experiment; but in this work the ratio $Q = B/A$ was usually determined in the daytime with magnetometer sensibilities of the order of one-tenth those used in the principal experiments. In some cases, for the sake of additional security, determinations were made with the greater sensibilities, but no appreciable differences were found. The ratio B/A was always determined from long series of large deflections, which were usually made approximately equal by making one of the currents an exactly known multiple of the other, and multiplying by the proper factor.

In determining the quantity $Q = B/A$ it is important, for precise work, especially when the upper absolute sensibility coil is used, to have the field in which the rotor moves approximately neutralized. This is necessary because *Q* depends on the ratio of the moments of the magnetometer magnets, and this ratio is different with and without compensation. The effects produced by compensation, however, are very slight, *Q* with and without compensation differing by only about one-third per cent.

22. **Process C. Reduction to uniform field. The reduction standards.** If the calibrating solenoid used in obtaining the rotor sensibility *B* were infinite in length, it would, for equatorial positions of the magnetometer, have *no effect* on the magnetometer and would produce a uniform axial field throughout the rotor, like that of the intrinsic intensity of rotation. The shortness of the solenoid actually used, as well as the work with axial positions of the magnetometer,

necessitated the third process C, in which for each rotor and position, so far as necessary, a determination was made of the ratio of C, the rotor sensibility as it would be if the solenoid were very long, to B.

For this purpose two other solenoids, the *reduction standards*, were constructed—one for equatorial positions and the other for axial positions. The axial standard was similar to the equatorial except that it was shorter, because it could reach only to the magnetometer. The equatorial reduction standard was more than 40 cm. longer than the primary, and had very closely the same diameter. Like the primary it was very uniformly wound with 36 turns per inch. The main terminals were arranged as in the primary solenoid. In addition, leads were soldered to the ends of a central section equal in length to the primary standard, and twisted together in the usual way.[29]

In the determination of C/B, or of $K = (B - C)/B$, a quantity more convenient in computation, two alternative processes were used. In the first, and ordinarily used process, in order to facilitate the determination, one of the astatic magnetometers was mounted on a separate stand with the same orientation as in the principal experiment, but with greatly reduced sensibility, and currents were used which were much larger than the calibrating currents, but not too large to prevent the magnetic moments from being strictly proportional thereto. As the solenoids were wound with the same pitch, and identical currents were used, the moments produced were nearly the same and therefore, in the equatorial positions, the deflections.

An adjustable stand was arranged to bring the rotor under investigation, centrally and axially mounted in the primary standard, and its compensator, into essentially the same positions with respect to the magnetometer system as those used in the principal experiments.

For each of the chosen positions, identical currents were passed in turn through the complete solenoid and its central part, and the corresponding magnetometer deflections were obtained on repeated reversals.[30] The absolute sensibility was determined, and the whole process was, in most cases, repeated (though this was found to be

[29] Equality of the number of turns per unit length in primary and reduction standards is of course unnecessary. Minute differences in diameter and length, if present, produce only differences of a higher order in the resulting moments, and are thus of no consequence.

[30] In some later experiments the process was much improved as follows: The equatorial solenoid was replaced by one with the same length of central section and (as it happened) greater pitch, *the whole being insulated from the tube*. Deflections were obtained when the current traversed the *terminal sections only* and when it traversed the *central section only*, with rotor inside, and when it traversed the complete solenoid alone.

unnecessary) when the rotor had been turned through one-third of a revolution about its axis, and again when it had been turned through two-thirds of a revolution (or through one, two, and three quarters of a revolution). Then the rotor was removed, the magnetometer sensibility again determined, and deflections obtained on repeated reversals through the complete solenoid of a current much larger than that used when the rotor was present, but bearing a known ratio to it.

In the other process, the procedure was essentially the same, except that the determinations were made with the magnetometer and rotor in their standard experimental positions in the compensating frame and with the sensibility of the same order as those actually used in the rotation work.

For a given position of the magnetometer and a given rotor, let d_1 and d_2 denote the mean deflections, reduced to the same magnetometer sensibility, obtained on reversal when the current traverses the complete solenoid and its central portion only, respectively, with the rotor inside; and let d_3 denote the deflection, reduced to the same magnetometer sensibility, obtained when the same current traverses the complete solenoid with the rotor removed. Then the deflection d which would be produced, for the same sensibility, by the rotor alone when magnetized by the uniform field of the complete solenoid is $d = d_1 - d_3$. Hence, since the effect of the central portion of the reduction solenoid is identical with that of the primary standard, $C/B = d/d_2$, or $K = (d_2 - d)/d_2$.

The quantity K as determined for all of the rotors in equatorial positions is given in Table I, column 5. Most of the experiments were made in the nearer equatorial positions, but in the several cases (including extreme cases) tested, no difference could be detected when the rotor was nearer to or more remote from the magnetometer. Nor did the removal of the compensator make any difference, nor the substitution of weaker for stronger fields. The observations for K were made with great care, and no one of the results given for equatorial positions is probably in error by as much as one unit in the first decimal place. Accurate observations (indicated by AF in the table), but not so accurate as those for the equatorial positions, were made for one axial position and with but few rotors; but rough observations, made with a simple magnetometer, showed no differences for different axial positions. Also, observations made on some of the rotors near the end of the investigation gave practically identical results with those made much earlier.

23. **Insulation of the standard coils.** The three secondary standard coils used in the course of the principal rotation work were

wound on insulating cores. After the completion of the work, two of these coils, including that used throughout Series B, were completely unwound and their resistances found (practically) equal to the resistances when wound. In the case of the remaining coil, the only part in which there was any possibility of a short circuit was unwound and tested, with the same result as for the other coils.

Of the two reduction standards used in the principal experiments to determine the end effects, and wound, as already stated, on metal tubes, the insulation of one was perfect, that of the other, tested after the experiments were completed, so nearly perfect as to introduce no appreciable error. It gave, moreover, identical end effects at the end of the rotation work and very much earlier.

Before the beginning of Series A the insulation of the primary standard was tested and found to be perfect, the end of the coil soldered to the tube being temporarily freed for the test. The resistance of the coil and its leads was then measured at intervals until just after the completion of Series B, and never showed a diminution below its value when the insulation was known to be perfect. Eight months after the completion of Series B, however, and after transportation through Panama to California, with no precautions in packing to protect the coil, the resistance of the coil was found to have fallen 1.6 per cent, and the defect located just at the insulated end. Suspicion that the insulation might have become defective, at least at times, during the rotation work led us to apply additional tests as follows:

(1) With coils whose insulation was known to be (practically) perfect, we have made three independent determinations of the quantity Q for Steel III; one from observations made for other purposes near the end of Series A, with the magnetometer in its standard position, two from observations made especially for this purpose. When reduced to the conditions existing in Series A and Series B, these observations give values of Q two-thirds per cent *less*, one-half per cent *less*, and two per cent *less* than the values obtained with the coil which was suspected and whose insulation they would, if the discrepancies were not accounted for by the errors in observation and reduction, indicate to be *more than perfect*.

(2) As a further check on the values of the constants Q obtained for the various rotors in Series A and B (and therefore on the insulation of the primary standard) we have calculated for these series the ratio, ρ, of Q for each group of experiments with other rotors (so far as data were available) to Q for Steel III, whose behavior was exceedingly constant; and we have made four more series of direct determinations of this ratio with coils known to be (practically) perfect. All the values of ρ are given in Table III.

TABLE III.

VALUES OF ρ = RATIO OF Q FOR EACH OF THE ROTORS TO Q FOR STEEL III.*

Rotor	Values of ρ					
	(1) Series A	(2) Series B	(3) Aug. 26, 1924	(4) Aug. 29–31, 1924	(5) Sept. 12, 1924	(6) Dec. 2-4 1924
El. iron II	——	0.986 1.004	0.982	0.982	1.016	0.985
Armco iron	0.712	0.711	0.714	0.710	——	0.715
Norway iron	——	0.795	0.777	0.775	0.790	0.778
Steel III	1.000	1.000	1.000	1.000	1.000	1.000
Steel IV	——	0.833 0.855	0.860	0.854	——	0.862
Steel I	——	0.760	0.767	0.761	——	0.764
Nickel III	——	0.928	0.935	0.938	——	0.942
Nickel I	——	0.739	0.763	0.757	——	0.761
Cobalt II	0.810	0.817 0.820	0.786	0.790	0.801	0.792
Heusler's alloy I	——	0.483	0.476	0.474	0.473	0.474
Heusler's alloy II	0.372	——	0.374	——	——	——
Permalloy	1.257	1.257 1.255AF 1.238	1.245	1.250	——	1.259
Hopkinson's alloy	——	0.442	0.440	0.440	——	0.444
Preuss's alloy	1.079	1.091	1.104	1.095	——	1.105
Bloch's alloy	——	0.697	0.689	0.692	——	0.696

* Magnetometer in equatorial positions except when otherwise stated.

All the values of ρ in columns (5) and (6) and most of those in columns (1) and (2) are probably correct to one-fifth per cent. The remaining values of ρ are probably correct to one-third per cent. Yet discrepancies much greater than these errors occur in the table; and the discrepancies between the individual values in columns (3)–(6) are in general as great as those between these values and those of columns (1) and (2), or between different values for the same rotor in column (2).

The only factors to which it seems possible to attribute the differences in columns (3)–(6) are the magnetization, which was small in (6), but not so small as in (1) and (2), and larger in (3)–(5), and the methods used for demagnetization. No systematic differences, however, are apparent, and the considerable differences which occur would not be expected to originate from these causes. It is apparent that some of the rotors are much more nearly constant in their behavior than others.

It is clear from the table that we could not, with any certainty, ascribe the differences occurring for the same rotors in columns (1)–(2), or the differences between the values in these columns and those in (3)–(5), to imperfection of insulation.

24. **Sets of observations.** In order to reduce the accidental errors due to the fluctuations of the earth's field and other causes it was of course necessary in the principal experiments to make many magnetometer readings and to make them on a regular time schedule, at equal intervals, as in the case of processes A, B, and C; and to eliminate various systematic errors (see below) it was necessary to spin the rotor alternately in opposite directions at the same speed.

In the later work observations were usually made in sets of twelve, sometimes twenty or more for low speeds. In the case of a set of twelve, the motor was started, alternately right handed (as seen from the east) and left-handed, at the beginnings of twelve successive minutes.[31] Thirty seconds[32] later the magnetometer scale was read and the motor switch then opened. The procedure was similar for sets of eight or ten rotations, as in some of the earlier work, and for the larger sets. To obtain the mean double deflection for the set, the second scale reading was subtracted from the mean of the first and third, the mean of the second and fourth from the third, etc., and the average taken. Examples are given in § 60.

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31 In a few cases other equal intervals were used.
32 In some cases 35^s intervals were used.

25. **The equation for experimental determination of λ.** Let H_r denote the intrinsic magnetic intensity of rotation of a rotor for the angular velocity N revolutions per second, and d the magnetometer deflection produced by reversal of the direction of rotation. Also let H_c denote the uniform magnetic intensity which would be produced throughout the rotor by a minute electric current I traversing the calibrating solenoid if infinite in length, and C the deflection produced by the reversal of the magnetization it would cause. And let B denote the deflection produced by reversal of the came current I through the actual calibrating solenoid with the rotor inside. Then we evidently have

$$\frac{H_r}{H_c} = \frac{d}{C} = \frac{d}{B\,(1 - K)} \tag{5}$$

Let E denote the open circuit e.m.f. of the calibrating cell, in volts; R_o the resistance in the circuit, in ohms (that of the connections, battery, and solenoid being negligible); and n the number of turns per centimeter length of the primary solenoid. Then

$$H_c = 4\pi n\, E/10R_o \tag{6}$$

Also let A denote the deflection produced on reversal of the current I through the absolute sensibility coils, so that $B = AQ$.

Let G denote the ratio (gear ratio) of the rotor speed N to the motor speed S, and let T denote the time of 500 revolutions of the motor, so that

$$N = GS = 500\, G/T \tag{7}$$

Then, since we may write, with an error which is negligible in all of our experiments,

$$1/(1-K) = 1+ K$$

we obtain in place of (1) the equation

$$\lambda = \frac{H_r}{N} = \frac{4\pi n}{5000\, R_o} \cdot \frac{EdT}{GQA}\,(1 + K)\ \frac{\text{gauss}}{\text{r.p.s.}} \tag{8}$$

which gives the experimental value of the intrinsic intensity of rotation per unit angular velocity. In nearly all of our experiments n was 14.71 turns per cm. (36.00 turns per inch), R_0 was 100,000 ohms, and E somewhat greater than 1.5 volt.

When the calculation was made from magneto voltmeter readings instead of chronograph readings an equivalent formula was of course used.

The resistance R_o was furnished by a coil in a standard box by

Leeds and Northrup, and agreed closely with standard resistances from boxes by Wolff and the Pyrolectric Co. The electromotive force E was furnished by two dry cells in parallel and was measured nightly with a Wolff potentiometer, calibrated at the Bureau of Standards, and a Weston cell which agreed to 1 part in 50,000 with another similar cell, whose e.m.f. was determined at the same Bureau. The only appreciable errors accruing in connection with equation (8), as applied to any rotor, come from d, T, Q, and A, and the last three are ordinarily negligible, or very small in comparison with the error in d.

26. **The currents in the compensating coils.** The current through the coils of the main compensating frame was supplied by a 60-volt, 10-ampere storage battery used for no other purpose while the actual experiments were in progress. The compensating value of the current, in the final arrangement, was about 1.383 ampere and was adjusted and read to about 0.0005 ampere by a Weston semiportable voltmeter and shunt and suitable variable rheostats. A Wolff standard one-ohm coil was also in the circuit, and the currentmeter was frequently, in the later work nightly, checked with a Wolff potentiometer calibrated by a Weston cell. The copper resistance in the circuit was nearly one-half of the whole, so that the current required some time to become steady. It often did not change appreciably, but also often changed as much as 0.001 ampere, rarely more during a complete set of observations, though the circuit was always closed long before these observations began.

A single 60-volt, 30-ampere storage battery supplied the current for the three subsidiary compensating systems, an arrangement of switches permitting each current in turn to be read by a single Weston portable ammeter. This ammeter and suitable variable rheostats made it possible readily to adjust and read the three currents with much more than necessary precision.

The proper compensating currents in the coils of the main frame and in the A coils and B coils were determined with inductor coils and a ballistic galvanometer with very heavy electromagnetic damping. The inductor coil used in most of the work, for the main frame and the B coils, is shown in Figure 22. The length of its two coils was about 26 cm. and the breadth about 7 cm. A few minutes sufficed to clamp the coils over any of the rotors, or to remove them.

By setting this coil with the planes of its turns horizontal, and then turning it with the rotor, through 180°, first in one direction and then in the other, and reading the galvanometer throws for currents slightly

above and slightly below compensation values, the frame current necessary to reduce the mean vertical intensity through the coil to zero was readily determined by interpolation with greater precision than necessary to set the ammeter to 0.0005 ampere.

After the frame current had been adjusted to the proper value the test coil was set with the planes of its turns vertical, and galvanometer throws obtained on its reversal through 180°. This was done first with the current in the *B* coils zero, then with the current 0.50 ampere, 1.00 ampere, or some other appropriate value; and the correct value of the current for compensation of the mean horizontal intensity found by interpolation. The proper *A* coil current was determined in the

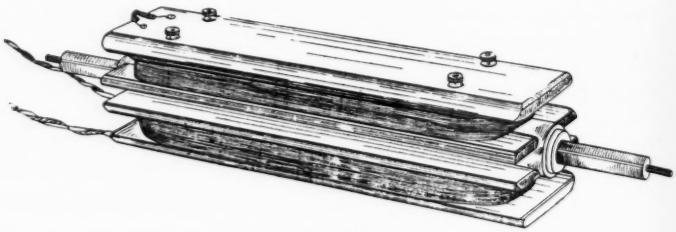

FIG. 22.

same way, but with a cylindrical inductor about 8 cm. in diameter and 6 cm. long mounted with its axis horizontal and so as to be movable about a vertical axis.

If we could assume the frame to remain entirely constant and the variometers to be entirely reliable, one set of determinations such as those just considered, together with the variometer readings, would suffice for the complete investigation. As two of the variometers, especially the vertical intensity variometer, were not trusted entirely, and as the frame (and for a time the windings) certainly underwent small changes, the compensation currents referred to (as well as the *C* compensation for nonuniform field) were determined on many occasions. In much of the work, in view of the insensitiveness of the magnetometer to eddy-current effects, the precautions taken were greater than necessary.

The proper setting of the current in the *C* coils was determined from observations on the rotation of a copper rotor, as described below.

27. **Position and adjustment of the compensator.** If the magnetometer is exactly astatic and the compensator and rotor magnetically exactly alike, it is clear that for the best compensation

they should occupy similar or symmetrical positions with respect to the upper and lower magnetometer magnets.

In much of the work the compensator was placed approximately as if these conditions were realized. Thus, in some of the work, it lay in *V* grooves, in the brass angle pieces *I*, Figure 3. Usually it was more exactly mounted, a closely fitting half-ring of brass, with 4 cm. external diameter, lying between compensator and *V*'s, which were so constructed that then the axis of the compensator was very nearly parallel, and in the same vertical plane, with that of the rotor, and the

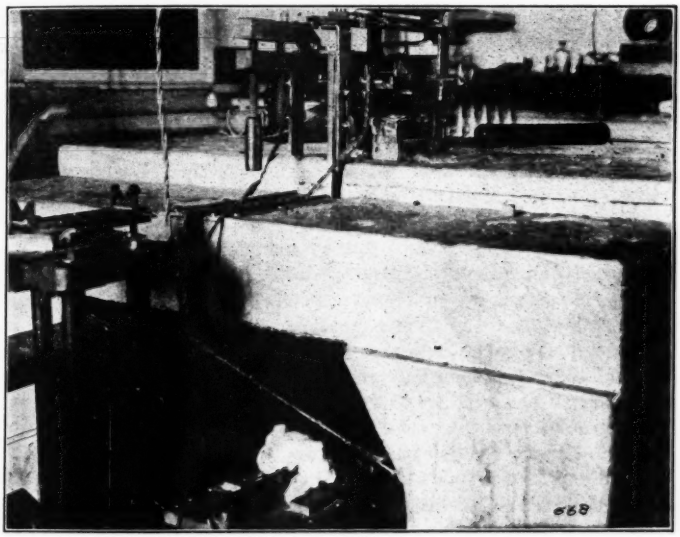

FIG. 23.

vertical distance between their axes very nearly equal to that between the centers of the magnet systems. The center of the compensator was placed vertically above the center of the rotor. Even when the rings were used, the arrangement was evidently not strictly correct for equatorial positions even on the assumption that rotor and compensator were just alike.

When the magnetometer was shifted on the casting *F*, Figure 3, in order to make the vertical equatorial positions possible, new arrangements had to be made for holding the compensator. The final arrangements are shown in Figures 6, 12, and 23. In the axial position

the compensator (not shown in the figures) lay in *V*'s about as far west of the upper magnet as the rotor was east of the lower. In the equatorial position the compensator was placed *above* the upper magnet and about as far from it as the rotor was from the lower magnet. Arrangements were provided for adjusting (equatorial position) in height, in axial distance from upper magnet (axial position), in altitude, and in azimuth.

These adjustments made it possible to correct very largely for (1) inequalities between rotor and compensator, for (2) inequality between the moments of the upper and lower magnetometer systems, and for (3) the horizontal angle between their magnetic axes. We shall describe the methods of adjustment for the equatorial position only. The same principles applied for the axial position.

Adjustments for (1) and (2) were made together. The frame holding the compensator, with its center in the equatorial plane passing through the suspension, and its axis approximately parallel to that of the rotor, was clamped in position. The magnetometer was then adjusted to the desired zero and sensibility.

A suitable current was then sent *in the same direction* through the two *C* coils, producing very nearly the same axial intensities in the rotor and compensator. If reversal of this current produced a deflection greater than the minimum desired, the height of the compensator was readjusted, and the process repeated until the deflection vanished or was reduced to the minimum sought. Fluctuations of the declination, or the east and west component of the horizontal intensity, then had little effect on the magnetometer.

In making the adjustment for (3) the *B* coils were used. With a suitable current traversing the coils and the magnetometer in adjustment, the deflection produced by reversing the current was determined. Then the azimuth of the compensator was altered, and the process repeated until the deflection on reversal vanished or was not greater than the minimum desired. The magnetometer was then affected but little by fluctuations in the north and south component of the horizontal intensity.

During the adjustments mentioned the rotor was ordinarily in slow motion, usually in a neutral field, so that the mean magnetic axis of the rotor coincided with its geometric axis, and the moments of the magnetometer magnets assumed the values they had in the rotation experiments. The adjustments for (2) naturally differed considerably when made in and out of the earth's field, while that for (3) was not affected in this way.

If the compensator is homogeneous and so leveled as to be parallel to the rotor, adjustments made as above are sufficient. If, however, the compensator is not homogeneous, or its magnetic axis (for induced magnetization) not coincident with its geometric axis, the adjustments here described are not sufficient to prevent fluctuations in the vertical intensity from affecting the magnetometer. Hence, for bad compensators, it was necessary to proceed as follows in adjustment (3): A divided scale was pasted around the compensator so that its position angle with reference to a fixed mark could be read. The position angle, sensibility, and deflection were then determined. Then the process was repeated for a number of angles through a complete revolution of the compensator, and the position angle at which the deflection (reduced) was a minimum or a maximum determined. The compensator was then set at one of these two angles, with its magnetic axis in the plane normal to the central line through rotor and compensator, and the proper azimuth found as above. The magnetometer was then insensitive to fluctuations of either horizontal or vertical intensity.

In making the adjustment (3) when the horizontal angle between the magnetometor magnetic axes was considerable, the compensator had sometimes to be turned out of parallelism with the rotor through a number of degrees. This, of course, magnetized the compensator and produced a magnetic field at the rotor. Crude experiments, however, were quite sufficient to prove that the maximum intensities produced in this way were entirely negligible in their eddy-current effects on the rotor.

In the actual practice of the adjustments we did not, of course, attempt to make one adjustment final before making the other approximately, but proceeded by successive approximations.

28. **Systematic errors.** Aside from insulation errors, discussed below (§ 23), and other errors in the standards, which were entirely negligible, the principal systematic errors which had to be investigated or overcome may be divided into two classes: (A) those with origin independent of the magnetization of the rotor; and (B) those with origin dependent on the magnetization of the rotor, either permanent or induced.

Errors in class A may arise from: (1) eddy currents in the rotor due to incomplete compensation of the uniform part of the residual magnetic field; (2) eddy currents in the rotor due to incomplete compensation of the nonuniform part of the residual field; (3) electric currents due

to thermal effects at the bearings; (4) electric currents due to thermal effects produced by the air driven against the bedplate, etc., by the moving rotor; (5) eddy currents and other electrical or magnetic effects in the motor, starter and other driving apparatus; (6) thermal effects on the magnetometer due to air thrown against it by the motion of the rotor; (7) vibrations of the air inside the magnetometer case, affecting the suspension differently for different directions of rotation (see § 50).

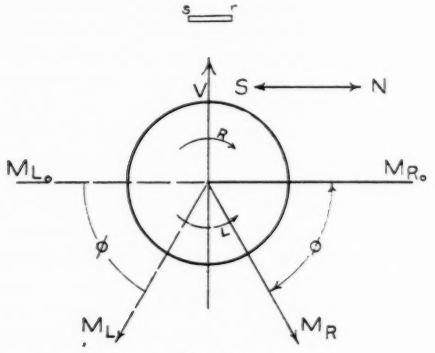

FIG. 24.

Errors in class B may arise from: (1) torsion of the rotor, which is driven from one end and moves with friction in a bearing at the other; (2) thermal effects on the rotor due to the bearings; (3) thermal effects on the rotor due to the air currents produced by its motion; (4) centrifugal expansion of the rotor or other strain, aside from torsion, produced by the rotation; (5) axial displacement of the rotor which may accompany its rotary motion; (6) displacement of the rotor in altitude or azimuth in the residual part of the field with intensity normal to its axis; (7) the Thomson repulsion effect on the magnetic system due to the irregularity of the rotor's magnetization; and (8) mechanical disturbances, different for right-and left-handed rotations.

29. **Eddy-current errors from residual uniform field.** Let us first suppose that the rotor is symmetrical and homogeneous and the residual field in which it moves uniform. The intensity of this field may be resolved into three rectangular components, L horizontal and parallel to the rotor's axis, H horizontal and normal to the axis, and V vertical and normal to the axis.

The component L produces no eddy currents, and only the horizontal components at the magnetometer magnets of the intensities h and v produced by H and V can affect the magnetometer. Let us denote the axial components of h and v by h_a and v_a, and their horizontal components parallel to the equatorial plane through the rotor by h_e and v_e.

The way in which the intensities h_a, h_e and v_a, v_e affect the magnetometer system, supposed always in the vertical plane through the axis

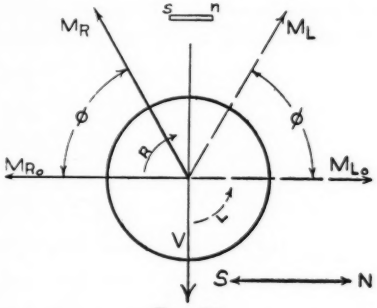

FIG. 25.

of rotation, can be seen from Figures 24–27, which give diagrammatic equatorial sections of the rotor and lower magnet when the magnetometer is vertically above the center of the rotor and which show diagrammatically the moments of the eddy-current systems to which similar right- and left-handed rotations give rise. The moments M_{R_o} and M_{L_o} are the moments which the eddy-current systems would have if there were no self-induction, and are 90° in advance of V and H; the actual moments M_R and M_L lead these vectors by an angle increasing with the speed.

30. **Eddy-current effects from residual uniform vertical intensity.** When H = 0, as in Figures 24 and 25, the vectors M_R and M_L, which are equal in magnitude, make equal angles with V on opposite sides, and their fields are symmetrical about the lines which represent them. Thus at any point in the vertical plane through the axis of rotation $(v_a)_R = (v_a)_L$, and $(v_e)_R = -(v_e)_L$.

The components v_e will affect the sensibility of the magnetometer, and oppositely for opposite directions of rotation, but will produce no deflection on reversal or other change in the rotor's velocity *unless* the

axes of the magnetometer systems are not parallel to the equatorial plane. For simplicity, we may neglect the effect (relatively minute in equatorial positions, and small in axial positions) upon the upper magnet system and consider only the lower, which is much closer to the rotor. If its axis makes an angle A (supposed small) with the equatorial plane, rotation will produce a deflection proportional to AV, changing sign with the velocity, so that it is doubled by reversal (*effect* α).

When the magnetometer is strictly in the equatorial plane, $v_a = 0$, so that it can produce no deflection. But if the magnetometer is given an axial displacement (supposed small) there will be, on changing the

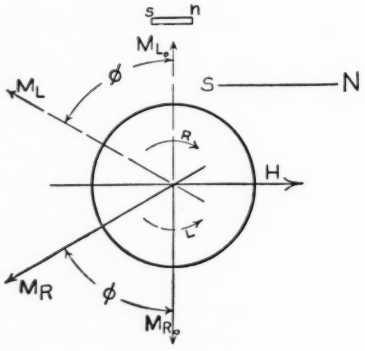

FIG. 26.

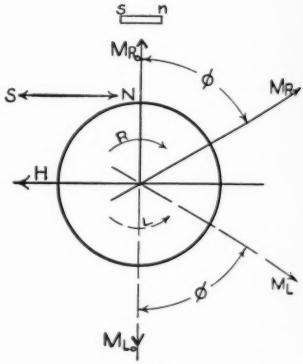

FIG. 27.

speed, a deflection (effect β) proportional to V and increasing with the axial displacement, but slowly on account of the small ratio of the diameter of the rotor to its length and the nearness of the magnet. Since $(v_a)_R = (v_a)_L$, there is no deflection produced by *reversing* the motion.

When the magnetometer is *axially* mounted, $(v_a)_R = (v_a)_L = 0$, and the effect β vanishes for *each* direction of rotation.

31. **Eddy-current effects from residual uniform horizontal intensity.** When $V = 0$, as in Figures 26 and 27, we find, in a similar way, that $(h_e)_R = (h_e)_L$. This component of the induced intensity alters the sensibility, equally for opposite directions of rotation, the alteration changing sign with the residual field intensity H, but produces no deflection unless the magnet system makes an angle

(A) with the equatorial plane. In this case there results a deflection (*effect* γ) proportional to AH, but independent of the direction of rotation.

In the equatorial plane, $(h_a)_R = -(h_a)_L = 0$, so that the magnetometer is not deflected by this component. But if the magnetometer is given a small axial displacement, there will ensue a deflection (*effect* δ) increasing with the displacement and proportional to H, and changing sign, without alteration of magnitude, on reversal of the rotation.

If the magnetometer is in an axial position the effect δ vanishes.

32. **Eddy-current effects due to residual nonuniform intensity.** If the residual *horizontal intensity* H is not constant through-

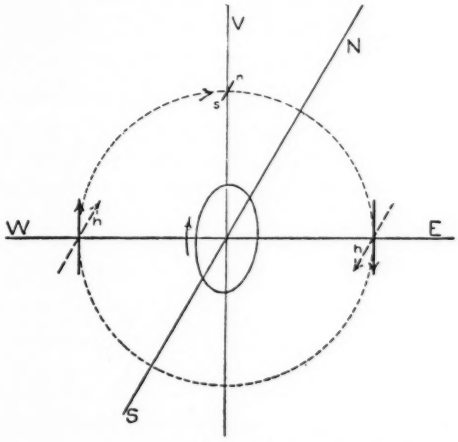

FIG. 28.

out the rotor, it may, with sufficiently close approximation to the truth, be resolved into two components, one, H_o, constant throughout the rotor, and equal to its value at the center, and the other, h, proportional to the axial distance from the equatorial plane, Figure 11.

The component H_o may be treated precisely like H in what precedes. For qualitative purposes the component h can be treated much in the same way, due attention being paid to the fact that its directions are opposite on the two sides of the equatorial plane. Figures 26–27 may then be applied roughly to each *half* of the rotor.

Figure 28 shows, for right-handed rotation of the rotor, and a given

distribution of h, the vertical components of the resulting moments of the two halves of the rotor and the direction of the field they produce at the lower magnetometer magnet in a vertical equatorial position. The horizontal components of the moments have evidently no effect on the magnetometer. Reversal of the rotor produces a deflection (effect ϵ) proportional to h and opposite in sign for the two directions of reversal.

The *vertical intensity* V, when not uniform, may be treated precisely like H, and resolved into two components V_o and v. Figures 24–25 may be applied roughly to each half of the rotor, and the effect of v may be seen through diagrams analogous to those of Figures 11 and 28. The component v produces deflections (effect ζ) which are independent of the direction of rotation, so that the deflection on *reversal* of the rotor's speed vanishes.

Effects ϵ and ζ both vanish in the axial positions of the magnetometer for each direction of rotation if the angle A is zero, as does the effect ϵ in any case, but the horizontal components of the fields associated with them may affect the sensibility, which they do not do in equatorial positions.

33. **Nonhomogeneity of the rotor.** If the rotor is not homogeneous, the two halves being unlike, what precedes will apply to the mean results obtained by observing first with one end of the rotor east, then with the other east.

Second order effects. In all that precedes effects, such as $\delta\alpha$ and $\delta\gamma$, due to the slight change of A produced by the rotations themselves are neglected. In our experiments they vanish or are eliminated with α, β, etc.

34. **Elimination of the eddy-current errors.** The process of eliminating the effect α consisted in making the uniform part of the field in which the rotor moved so weak and the angle A so small as to render the effect negligible.

Two general methods of testing for and reducing the angle A were used. In one, a small and accurately made solenoid with horizontal axis was mounted with its center vertically beneath the magnetometer suspension. The solenoid was turned about a vertical axis until its (horizontal) axis lay east and west. A steady electric current was then sent through the solenoid and the deflection produced on reversing its direction was determined. The process was then repeated, but with the solenoid's axis north and south. From the two deflections the angle A could be determined, and then reduced as much as desired by the zero control.

In the other method, a copper rotor was sometimes used. With the main compensating current considerably increased or decreased from its normal value, right-handed and left-handed rotations were produced in the usual way in sets, and the mean deflection due to reversal obtained. The zero was then altered and the process repeated. From the two groups of observations the proper zero could then be obtained by interpolation. The same method was also applied by using magnetic rotors, the zero being adjusted until the deflections were nearly the same for under-compensation and over-compensation much greater than those occurring during the principal experiments.

The error due to the effect α is opposite for axial and equatorial positions, and can therefore be eliminated, or its limits determined, by using both.

The effect δ in the weak residual field was made negligible or very small by setting the magnetometer very nearly in the equatorial plane, or by setting it in an axial position. The effect ϵ was made negligible by compensating the nonuniform part of the residual horizontal field by means of the coil system C, described above. Both effects were tested for by rotating both copper and the magnetic rotors, the adjustments being such, in much of the recent work, that changes in the currents of the B and C coil systems corresponding to residual fields much stronger than could exist in the rotation experiments produced only negligible effects.

35. **Eddy-current observations on copper made with the compensating system of the principal experiments.** We have made many sets of observations on a rotor of copper about 3.1 cm. in diameter and length similar to that of the other rotors and similarly placed. If we assume the conductivity of copper to be eight times that of iron, and the rotors of the same diameter and infinite in length, in a uniform field normal to their axes, and if we neglect self-induction, it is easy to show that the effect α of the rotor eddy currents on the magnetometer in equatorial positions is about four times as great for copper as for iron. In general the eddy-current effects with copper are much greater than with the magnetic rotors.

In Table II, A is the absolute sensibility; d the deflection obtained when the copper is rotated, and the direction of rotation reversed, in a field whose uniform part, and that only, is supposed to be annulled (except for the second line for May 16, 1923, which refers to observations made with the C current properly adjusted to reduce the effect ϵ to zero, and for the observations of December 28, made with the B cur-

rent too large by 13 divisions); $\Delta d/\Delta V$ is the increase in deflection produced by increasing V (the current in the main compensating frame) by one division (0.01 ampere), usually from half a division below the compensating value to half a division above; $100\Delta d/\Delta C$ is the increase of deflection produced by changing the C current 100 divisions (1.00 ampere) in the positive direction; and $50\Delta d/\Delta B$ is the increase of deflection produced by increasing the B current by 50 divisions (0.50 ampere).

TABLE IV.

RESULTS OF OBSERVATIONS ON COPPER, AUGUST 9, 1922 TO DECEMBER 28, 1923.*

All deflections in centimeters. Gear ratio 2−1.

Date	Mag. pos.	No. sets	Approx. A	d	$\Delta d/\Delta V$	$100\ \Delta d/\Delta C$	$50\ \Delta d/\Delta B$
1922							
Aug. 9	*LEQ*	6	17.4	*minute*	+0.04	——	+0.04±0.02
1923							
Mar. 6	*LEQ*	6	17.3	−0.28 ±0.03	+0.19	+0.27 0.02	——
Mar. 13	*AN*	4	17.6	−0.01 0.05	——	−0.03 0.05	——
†May 8	*LEQ*	7	18.2	−0.02 0.08	−0.10	——	——
‡May 16	*LEQV*	3	18.4	+0.16 0.04	——	+0.45 0.03	——
‡May 16	" (C=−31)	1	18.4	+0.03 0.06	——	——	——
Aug. 24	*LEQV*	4	17.3	+0.19 0.04	−0.58	+0.34 0.03	——
Oct. 19	*EQV*	4	18.4	+0.07 0.04	——	+0.26 0.04	——
Nov. 11, 14	"	10	18.0	+0.06 0.07	——	+0.29 0.04	——
Dec. 20	"	4	21.2	+0.03 0.06	——	+0.41 0.03	——
Dec. 28	"	2	16.1	+0.02 0.07	——	+0.17 0.04	——

* Some observations made on Nov. 19, 1922, with poor adjustments, are not included in the table.
† Single longitudinal loop in use.
‡ Double longitudinal loop in use.

From the value of A and that of $100\Delta d/\Delta C$ the proper value of C can be calculated on the assumption that d is wholly due to the nonuniform part of the horizontal intensity. Thus from $d = +0.16$ for May 16, 1923, and $100\Delta d/\Delta C = +0.45$, we find the proper value of C to be -31 divisions. When C on this date was made -31 divisions, d became very small, viz., $d = +0.03 \pm 0.06$.

The assumption that d is wholly due to the nonuniform part of the horizontal intensity is not strictly justified for the position *LEQ*, in

which the nonuniform part of the vertical intensity is also effective, since the suspension is not in the vertical plane through the axis of the rotor.

36. **Further eddy-current experiments on copper and magnetic rotors.** In other special experiments on the rotors of copper and annealed steel we have confirmed the deductions of §§ 29 and 30 with reference to the effects of rotation on magnetometer sensibility, and have determined the effects β and δ in vertical and horizontal fields normal to the rotor and equal in magnitude to that produced by 100 divisions of current in the B coils. The experiments were made with the aid of a special compensating frame in relatively strong fields and at low sensibilities, and the results then reduced to those which would have been obtained with the intensity indicated above and the approximate absolute sensibility $A = 20$ cm.

In Table V β and δ are the increases in the deflection due to reversal of the rotation when the magnetometer is shifted through the central position 1 cm. toward the east, the first for a field in which the intensity is vertical and downward, the second for a field in which the intensity is horizontal and northward. The quantity $\Delta A/A$ in the last column is the change in the absolute sensibility produced by the rotation of the copper, either right-handed or left-handed, in the northward horizontal field (with intensity equal to that produced by a current of 100 divisions in the B coils). Changes of similar magnitude, but opposite in sign for the two directions of rotation, were produced by a vertical field, which produced effects not certainly appreciable on the steel rotors.

TABLE V.

EDDY-CURRENT EFFECTS IN COPPER AND STEEL. MAGNETOMETER POSITION *EQV*.

Rotor	Speed	β	δ	$\Delta A/A$
Copper	26 r.p.s.	−0.02 cm.	+0.92 cm.	—
	61	+0.04	+1.09	$-\frac{1}{3}\%$
Steel III	26	+0.09	+0.18	—
	61	+0.02+	+0.22	—
Steel IV	61	−0.06	+0.09	—

The effect β, for reversals of rotation, should be zero, and the departures from zero in the table are doubtless accidental. The effect δ for copper, as would be expected, is much greater than for the other rotors.

The copper rotor (like the others) in these experiments was in its final state. Its homogeneity is proved by the fact that in a number of sets of observations reversal from mark east to mark west produced (in the horizontal field indicated above and with $A = 20$ cm.) only the deflections $+ 0.035 \pm 0.03$ cm.

The effect δ, as the table shows, is only slightly greater for the higher than for the lower speed, a fact which emphasizes the importance of working at the higher speeds in the principal experiments.

In the principal experiments in equatorial positions the axial displacement of the magnetometers center from the vertical plane passing normally through the center of the rotor was small, very minute in Series A, and never greater than about 2 or 3 mm. in Series B. The greatest value of ΔB, the excess of the B current over its correct value for exact compensation, viz., +18 divisions (see Table XIII), occurs with the Norway iron rotor, whose conductance (Table I) is about equal to that of Steel III. The double deflection for this rotor due to the effect δ at the highest speed would be, at the greatest, about $\frac{127}{122} \times \frac{3}{10} \times 0.22 \times \frac{17.4}{20} \times \frac{18}{100} = 0.01$ cm., which is much less than the accidental error. And the effects δ are smaller, in most cases much smaller, for the other rotors in Series B. The effect vanished for all the magnetic rotors in Series A.

Many additional observations on eddy-current effects in the magnetic rotors made with the compensating system used in the principal experiments are given in Table IX, § 56. We give here a few results, obtained with the same compensating system, on the variation of sensibility with speed on account of incorrect compensating currents.

With Permalloy, and the magnetometer in the position AN, the sensibility was determined at the gear ratio 2−1, for right and left-handed rotations, the first giving $A = 20.65 \pm 0.08$, the second $A = 20.86 \pm 0.18$, when V was 0.5 division too small. The sign of the difference of 1% is in accord with the theory, the vertical components of the residual intensity in this case acting to diminish the sensibility for right-handed rotations, and to increase it for left-handed rotations.

With Steel III, magnetometer position $LEQV$, and the B current 40 divisions too small for correct compensation, very slow rotation gave $A = 20.18 \pm 0.09$; while right-handed rotation at the gear ratio 2−1 gave $A = 20.30 \pm 0.16$. Again the sign of the small effect is in

accord with theory. The result is larger than would be expected from the above results with the copper rotor, but a considerable part of the apparent effect may be due to accidental error.

37. **The effect of the lower magnetometer magnet on the compensation currents.** By means of simple induction experiments, involving the use of a magnet whose moment was a considerable and known multiple of that of the lower magnetometer system with six magnets, and which it is unnecessary to describe in detail, we determined for a number of different cases the flux Φ which this lower magnet sent through the coils of Figure 22 clamped over a rotor.

For the position *LEQ*, a copper rotor, and the planes of the turns *horizontal*, Φ (vertical) was equal to the flux produced by the frame current corresponding to 0.035 division on the ammeter, the vertical flux of the uncompensated field corresponding to about 138.5 divisions. With the planes of the turns *vertical*, Φ (horizontal) was equal to the flux produced by the frame current 0.011 division, the horizontal flux of the uncompensated field corresponding to about 138.5 divisions.

For the position *LEQV*, and a large nickel rotor, the vertical and horizontal fluxes were respectively 0 and that produced by an intensity 0.00088 *H*, corresponding to 0.077 ampere in the *B* coils.

For the position *EQV*, the vertical and horizontal fluxes were 0 and that produced by an intensity 0.00065 *H*, corresponding to 0.057 ampere in the *B* coils.

For the two axial positions used, the vertical fluxes were of course 0, and the horizontal fluxes for the positions *AF* and *AN* were about 0.1 and 0.2 that for *EQV*.

The above values are all for a lower system with six magnets. For a lower system with three magnets the fluxes are only about one-half as great.

38. **The effect of the control magnets and upper compensating coils on the compensation currents.** The small upper compensating coil, when traversed by the compensating current usually employed, viz., about 1.4 ampere, had a moment of about 10 c.g.s. units.

This coil thus produced at the axis of the rotor, about 60 cm. below, a normal horizontal intensity of about $H/4000$, symmetrical about its center.

The control magnets, whose moments are given in § 8, produced intensities which were very much less.

39. **Eddy-current effects of the lower magnetometer magnet.** When in the position *EQV*, a six-magnet system, 12.9 cm. above the rotor's axis, and normal to it produced at the center of the rotor a horizontal intensity about $0.00094 \times H$, a three-magnet system about $0.0005 \times H$. On account of the rapid falling-off of the intensity toward the ends of the rotor, this field, from symmetry, could give rise to only a minute effect δ even if its intensity were greater and the magnetometers were considerably displaced from the central position.

If the axis of the magnet is not strictly normal to the rotor's axis, the axial component of its moment produces along the axis of the rotor a nonuniform field whose vertical component for a special case is given in a curve (marked "6 magnet system," etc.) of Figure 13. If this field were *fixed*, and the magnet free to move, no change of magnetometer reading on reversal of the motion would occur. Inasmuch, however, as the angle changes, on account of the rotation effect under investigation, on reversal, there is a second order effect produced. From Figure 13 and the fact that the axis of the lower magnet points north, it is easy to see that effect is such as to *reduce* the deflection produced by reversal of rotation.

40. **Other effects of the lower magnet.** If the lower magnet, six-magnet system, distant 12.9 cm. from the rotor's axis, makes an angle θ with the equatorial plane, it produces at the center of the rotor an axial intensity $h = 0.00094 \times H \times \theta = 0.00018 \times \theta$ gauss; and if the angle changes by $\Delta\theta$, $\Delta h = 0.00018\Delta\theta$ gauss. For a 4-cm. deflection $\Delta\theta = \frac{1}{2} \times 4/600$ (approx.) $= 0.0033$; and $\Delta h = 6 \times 10^{-7}$ gauss.

If a 4 cm. deflection is produced by reversing the rotation at a speed 60 r.p.s., the change $\Delta h'$ in the intrinsic intensity of rotation is about $3.7 \times 10^{-7} \times 60$ gauss, or about 220×10^{-7} gauss. Thus Δh is less than 3% of $\Delta h'$.

Suspicion that Δh, as well as magnetometer eddy-current effects, might be sufficient to affect our results appreciably led to experiments in which the magnetometer sensibility was greatly reduced (ratio $\frac{1}{2}$), the six-magnet systems replaced by three-magnet systems, and the position of the magnetometer altered by increasing its height 4.7 cm. All were without appreciable effect. In theory Δh is fully taken account of in the calibration.

41. **Elimination of errors due to thermal currents from bearings.** When the bearings supporting the rotor and various countershafts are not electrically insulated from one another, electric

currents of thermal origin are in general produced by the rotation, and they produce a systematic error if the heating of the journals depends on the direction of rotation, as it may do even when the opposite angular velocities are equal. We have definitely observed the objectionable effects of such currents only in the case of the copper rotor, with which it is easy to recognize them as of thermal origin by their time characteristics. This effect has been avoided in all the recent work by sufficient insulation of all the bearings within a considerable distance of the magnetometer.

Furthermore, when all the bearings were insulated, and the copper was rotated in a weak residual field by a half-inch journal in a bearing of lignum vitae in the east bearing block, we obtained (January 11, 1923) from many rotation reversals at about 60 r.p.s. and with the magnetometer in the lower equatorial position the deflection $d = -0.02 \pm 0.08$ cm.; but when the wood was replaced by lumen metal, we obtained $d = +0.19 \pm 0.13$ cm.; with absolute sensibility A about 16.0 cm. The errors are large, but there is apparently evidence of a definite effect of failure to insulate one journal from its bearing. Any such effect was eliminated or greatly reduced by the type of journal and bearing finally adopted and already described.

42. **Effect of air currents on bedplate.** In seeking to account for some of the observed discrepancies, we were led to suspect that the air currents produced by the rotation of the rotor and impinging on the (bronze) bedplate differently for the two directions of rotation might produce a differential deflection on reversal. The rotations in Table VI on copper, which were obtained while investigating the effect ϵ, possibly show such an effect; while the somewhat better observations made with a still larger rotor of wood, for this purpose only, do not show the effect. In each case, some of the observations were made with a thick piece of cardboard between rotor and bedplate, and some with the cardboard removed.

TABLE VI.

(MAGNETOMETER POSITION *EQV*.)

Rotor	No. sets	Cardboard	Deflection	Sensibility
Copper	4	on	0.51 ± 0.06 cm.	18.1 cm.
"	6	off	0.65 0.09	18.1
Wood	3	on	-0.02 0.08	18.0
"	3	off	-0.02 0.06	18.0

Other experiments, with various rotors, in which rotations were made with and without the brass calibrating tube about the rotor, or with a semicylindrical piece of cardboard, designed to separate the rotor from the magnetometer but not from the bedplate, in place and removed, do not show any effect of the screening.

This effect will be referred to later in connection with the error *B* (3), § 28.

In most of the final work the rotor was screened from the bedplate by heavy cardboard or was completely surrounded by a thick cardboard tube about 6.2 cm. in diameter (internal). See Table XIII.

43. **Elimination of thermal effects on magnetometer.** It has long been known, or at least suspected, that small quantities of heat transferred to a sensitive magnetometer (at least a magnetometer with a metal case) may produce deflections which are far from negligible; and we have obtained much evidence for the existence of such an effect. We of course suspected the possibility of a systematic error from this source in this investigation, since the rotor, if it throws the air against the magnetometer, does so differently for the two directions of rotation. The error would evidently be much greater for equatorial than for axial positions of the magnetometer. The thorough wrapping of all but the upper part of the magnetometer with cotton and paper apparently sufficed (if indeed it was necessary) to prevent this effect, even without the tube or cardboard screen with which the instrument was separated from the rotor in nearly all but the early work.

44. **Errors due to the motor and other driving apparatus.** Table VII, *A*, gives the results of the tests which were made, with the field of the earth at the center of the frame annulled, on three single-phase motors which were used in work with the large compensating frame. The motor and gear box, when in the position marked "in," were inside the building, just inside the east end of the frame; in the position marked "out," they were in the pit east of the building and about twice as far away from the magnetometer. *A* is the magnetometer sensibility, d the mean magnetometer deflection $(R-L)$ produced on reversal of the motor, and $\Delta' d$ the average departure of this mean from the mean deflections for all the sets in the group.

The observations showed no certain differences for the gear ratios 2−1, 1−1, and 0 (motor and one gear shaft running alone); nor for different rotors (with their compensators), soft iron, nickel, and none. Hence the results for each motor and magnetometer position are all grouped together.

In these observations the rotor, when used, was clamped in position, and the small chuck and rod by which it was ordinarily connected to the first countershaft were removed. Many experiments have shown that in the compensated field this small element produces no appreciable deflection. When the compensating current was annulled, a deflection of + 0.84 cm. was obtained at the speed 2 − 1.

Table VII, *B*, gives the results with the three-phase motor in compensated fields. The first group was obtained from August, 1922 to January, 1923; the second in May–June, 1923; the third in October–November, 1923. In some cases there was no rotor (or compensator) present; in others there was a rotor of wood, either at rest or in rotation; in others, rotors of copper, nickel, permalloy, or cobalt, always at rest (with their compensators); in some cases the chuck and connecting rod were removed, while in all the later work they were present and in rotation. The observations show no differences depending on these conditions, as they should not if the motor and other very remote parts of the rotating system are alone responsible for any effect obtained. The quantity $\Delta' d$ is the average departure of the individual observations in a set from the mean for the set, averaged for all the sets in the group; while the quantity $\Delta'' d$ is the average departure from the group mean of the means for the sets which constitute it. The effect is evidently zero within the limits of the experimented error for all speeds.

When the earth's field at the center of the frame is not annulled, and eddy currents are developed in the near parts of the system, the deflections are very different for different conditions — ranging, for the gear ratio 2−1, from + 1.4 cm. with a wooden rotor and the magnetometer position *LEQ* to −0.1 cm. for the permalloy rotor and the position *EQV*, with the connecting rod and chuck in rotation in both cases.

In obtaining the later results in compensated fields, in which the rotors were always clamped in position, it was of course possible that residual cross-magnetization of the (resting) rotor might induce currents in the near-by chuck and driving rod. Hence the experiments were made in pairs with the position angles of the rotor differing by 180°, and the means taken. No certain effects of the rotor's magnetization were noticed.

TABLE VII.

EFFECTS OF DRIVING APPARATUS, WITH SINGLE-PHASE AND THREE-PHASE MOTORS.

A. Repulsion Start Single-phase Motors.

Motor	Magnetometer position	Motor position	No. sets	*A*	*d*	$\Delta' d$
1½ h.p.	*LEQ*	in	8	16.2	−0.18	±0.03
1½ h.p.	*AF*	"	8	18.4	−0.13	0.07
3 h.p. No. 1	*LEQ*	out	6	18.7	+0.06	0.02
3 h.p. No. 2	*LEQ*	"	8	18.3	+0.06	0.04

B. 5 h.p. Three-phase Slip-ring Motor.

Speed	Magnetometer position	Motor position	Group	No. days	No. sets	*A*	*d*	$\Delta' d$	$\Delta'' d$
2–1	*LEQ*	out	1	4	12	18.9	−0.015	±0.02	±0.02
	LEQV	"	2	3	6	19.7	+0.02	0.02	0.03
	EQV	"	3	2	10	18.3	0.00	0.03	0.02
Means for 28 sets 2–1						19.0	0.00	±0.02	±0.02
1–1		"	2	2	4	19.7	+0.03	±0.04	±0.01
1–2	*LEQV*	"	2	2	3	19.7	−0.06	—	±0.06
	EQV	"	3	1	4	18.3	0.00	±0.02	—
Means for 7 sets 1–2						19.0	−0.03	—	—
2–7	*LEQ*	"	1	1	3	18.9	+0.01	±0.02	—
	LEQV	"	2	3	6	19.7	−0.04	0.02	±0.07
	EQV	"	3	1	4	18.3	+0.01	0.02	—
Means for 13 sets 2–7						19.0	−0.01	±0.02	—

45. **Demagnetization and reversal of rotors and compensators.** It is clear that all errors of class B would vanish with the magnetization of the rotor, and would be less the smaller the magnetization. They are eliminated more readily the more stable such magnetization as is residual.

In demagnetizing the rotors and compensators we have used the method of reversals and the method of mechanical shock. We have found that the stability of the residual magnetization is greatly increased if, in the application of the method of reversals, the current for each step is allowed to flow for a considerable interval. We have used, in all the recent work, intervals of about half a minute, with rather quick reversals between them, and very numerous steps. A great deal of time was saved by making the apparatus automatic. Sometimes we have succeeded in reducing the magnetization nearly or quite to the minimum readable with the testing magnetometer arrangement used by the method of reversals alone; but this was not usually the case, in spite of repeated attempts to perfect the apparatus. Most of the magnetization left after a long application of the method of reversals, with the current usually reduced to zero by hundreds of steps, was usually removed by striking the rotor, with a block of lead weighing about half a kilogram, more or less sharply and with its magnetic axis making a greater or less angle with the intensity of the earth's field, according to the rotor's characteristics.

In order to keep the magnetization as nearly constant as possible, both when the magnetization was being tested and when the rotor (or compensator) was being reversed in the principal experiments, reversals were always made, except in the earliest part of the work, with the axis of the rotor (or compensator) turning in a plane normal to the intensity of the earth's field.

The intensity of magnetization of the rotors as used in the last series of experiments is given in column 30 of Table XIII. For the preceding series, the rotors were in general still better demagnetized. The magnetization of the compensators was always so small as to have entirely negligible effects on the field in the region occupied by the rotor. A measure of the irregularity of the residual magnetization of the rotors as used in the last series is given in column 31 of Table XIII, where the "oscillation" is approximately equal to the range of motion of the spot of light on the magnetometer scale which would be produced by very slow rotation of the rotor in the compensated field.

46. **Elimination of errors due to torsion.** In the investigation of 1914–15 on large rods of cold-rolled steel the effects of torsion, due to journal friction at the end of the rotor remote from the motor, were found to be negligible. Also in the early magnetometer work with cobalt and nickel our tests did not reveal any certain torsional effect. In the recent work, however, in which the range and precision of the

observations have been greatly increased, torsional effects have often appeared.

Torsional effects are of two kinds, according as the magnetization is residual or induced. The first vanishes with the residual magnetization, the second with the residual axial field.

For very small twists, at least, the first is proportional to the torque, and thus changes sign with the direction of rotation; the second is proportional to the square of the torque and independent of the direction of rotation. The first effect may thus give rise to a systematic error. The second, provided the magnetization is stable, cannot do so if the principal observations are made only for equal speeds in opposite directions, unless the journal friction at the remote end of the rotor depends on the direction of rotation.

We have no evidence of the appearance of the second effect in our recent work, and many experiments have given the same results with and without an axial field.

If the two journals are exactly alike, the rotor balanced, and its magnetization stable, the first effect can be eliminated by observing first with the magnetic axis of the rotor in one direction, then with the axis turned through 180° by reversing the rotor in its bearings. Both effects can be eliminated by direct measurements upon the change of the magnetic moment of the rotor due to the application of a known torque, and the determination of the actual frictional torque acting upon the journal remote from the motor in the rotation experiments. When the journals and bearings are similar, this is just one-half the torque driving the rotor, which can be determined electrically. Both torsional effects can be eliminated simultaneously by driving the rotor alternately from each of the two ends, one end always being free except for journal friction, provided the journal torques are identical at the free ends.

The first method is the one which we have almost always used. We have tried the second in a number of cases, but since all other friction in addition to the friction of the rotor bearings introduces irregularities, the method is not satisfactory with the delicate rotors used in this investigation. The third method we have not tried.

It is of course difficult to be certain that the two journals are exactly alike. The frictional torque should be made as small as possible, and then the effect of residual irregularities will be reduced. It was for this reason that accurate eighth-inch stellite journals, well polished, and running in an agate bearing with clock oil as lubricant, were substituted, at the west end of the rotor, for our original half-

inch journals. This effected a very great improvement. So far as torsion is concerned, the final three-sixteenths inch bronze journals and agate bearings appeared to perform equally well.

Inasmuch as the journal friction is nearly independent of the speed, the systematic error produced by torsion, when existent, is much less at high than at low speeds.

47. **Error due to thermal effects of journal friction on magnetization.** In addition to the thermal effect $A(3)$ which may produce a greater deflection with a magnetic rotor than with a non-magnetic one, the heat developed by journal friction may appreciably change the magnetic moment of the rotor, and differently for the two directions of rotation, and thus introduce systematic error.

So far as temporary magnetization is concerned, this effect vanished in our work, as no change was produced by the creation of even a considerable axial field.

As to permanent magnetization, if the journal torques for a given speed are identical at the two ends, the thermal effect of a systematic speed difference for the two directions of rotation can be eliminated by reversing the rotor in its bearings.

To reduce this error as far as possible, we first had the half-inch and eighth-inch rotor journals and bearings made as well as we could, and finally substituted the three-sixteenths inch bearings and insulated journals. This not only reduced the thermal effects at one end of the rotor, but greatly cut down the rate at which the heat produced was conducted to the rotor. Moreover, as the new journals were very precisely made, and as nearly alike as possible, the elimination of the error from any residual effect by reversal of the rotor was more nearly complete.

48. **Error due to thermal effect of air acting on rotor.** A very similar systematic error may be introduced by the air currents due to the motion, which, on account of asymmetry, may act differently on the rotor for the two directions of rotation. By reversing the rotor in its bearings this error also can be eliminated, provided the magnetization is sufficiently stable.

49. **Examples of torsion, journal friction, and air friction effects on magnetization.** In general these effects were made small by suitable demagnetization and by the other devices used. The following examples of the behavior of nickel show how troublesome they may be, also how they can be largely eliminated by reversal, even when of unusual magnitudes.

On October 16, 1922, a rod of nickel, through which the central residual flux density was about 0.04 line/cm.^{2}, was rotated at the gear ratio 2−1 with an eighth-inch bronze bearing west and a half-inch lumen bearing east. When the positive pole was *west*, right-handed and left-handed rotations *both* produced large deflections *up* the scale, the mean deflection being $\Delta = +21$ cm.±, but the differential deflection $d(= R - L) = +3.26$ cm. When the rotor was reversed, the positive pole being now *east*, the mean deflection was near $\Delta = -23$ cm.±, and the differential deflection $d = +3.60$ cm. The means for positive pole east and positive pole west are $\Delta = -1$ cm.± and $d = +3.43$ cm. (All deflections are reduced to the same magnetometer sensibility $A =$ about 17.8 cm.).

When half-inch bakelite bearings were substituted for the half-inch lumen bearing and the eighth-inch bronze bearing, and machine oil was used in place of clock oil (with which the bearings were lubricated in the experiments just described), the following results were obtained (for $A = 17.8$ cm., as above) with the same speed as before: With the positive pole *east*, $\Delta = -30$ cm., $d = -1.88$ cm.; with the positive pole *west*, $\Delta = +27$ cm. ±, $d = +8.63$ cm.±. The means are $\Delta = -1.5$ cm. ±, and $d = +3.38$ cm.

The observations for Δ were exceedingly rough.

With another rotor of nickel, of about the same dimensions, but provided with the latest type of insulated journals running in three-sixteenths inch agate bearings, and with residual induction about 0.07 line per cm.^{2}, we obtained, in November, 1923, for the gear ratio 2 − 1, the results $\Delta = -6.6$ cm., $d = +3.16$ cm. with the positive pole *west;* and $\Delta = +6.6$ cm., $d = +3.29$ cm. with the positive pole *east;* means, $\Delta = 0.0$ cm., $d = +3.22$ cm. All are reduced to the sensibility $A =$ 18.4 cm. (These values are not comparable with those given above, as the magnetometers used and their positions were different.)

The effect of the air on the rotor's magnetization is well illustrated by observations made at the same speed on the same nickel rotor shortly after the observations just described. A stiff paper bag was put over the lower part of the magnetometer, in such a way as to lie quite unsymmetrically with the vertical plane through the rotor's axis. For positive pole west we now obtained $d_W = +3.04$ cm.; for positive pole east, $d_E = +3.42$ cm.; mean, +3.23 cm., all reduced to the same sensibility as above, viz., $A = 18.4$ cm. As the rotor's magnetization and the speed remained nearly the same, the difference between the new value of $d_E - d_W$, viz., +0.38 cm., and the old value, without the bag, viz., +0.13 cm., must be attributed almost entirely to the change in the symmetry of the air effects.

50. **The errors due to centrifugal expansion and other strains aside from torsion.** The possible systematic error due to the change in the magnetic moment of the rotor brought about by centrifugal expansion, and shown in 1915 to be proportional to the square of the speed, can be eliminated, as formerly, by observing for equal speeds in opposite directions. If there is a systematic difference between the speeds for the two directions of rotation, error due to the permanent part of the magnetization can be avoided by observing for both directions of the magnetic axis of the rotor.

If there is axial induced magnetization, however, any error arising from it and the speed differences cannot be eliminated in this way. As already stated, the application of considerable axial intensities in the work described here was without effect.

It is, of course, possible that a *part* of the deflections described in the last article had their origin in centrifugal expansion.

Possible errors due to strains arising from lack of axial symmetry are eliminated precisely like those due to centrifugal expansion.

51. **Errors from axial displacement of the rotor.** A certain amount of axial displacement is always possible when a rotor is set into motion, and if the displacement depends on the direction of rotation a systematic error may thus be introduced. This may happen in three ways: (1) eddy currents due to the residual field will affect the magnetometer differently for the two directions of rotation from the way they would if the rotor were axially fixed; (2) the effect of the permanent magnetization of the rotor on the magnetometer will depend on the direction of rotation; and (3) if the axial field does not vanish or is not uniform or symmetrical about the equatorial plane through the magnetometer, the temporary magnetization of the rotor, and its effect on the magnetometer, will depend on the direction of rotation.

In the final arrangement of our apparatus the maximum possible axial play of the rotor was 0.05 mm., but the actual displacement which occurred when the direction of rotation was reversed was only 0.01 mm., and was identical for all the gear ratios $2-7$, $1-2$, $1-1$, and $2-1$. During the steady rotations there was a minute axial vibration present, but the mean position of one end of a small journal, which protruded through the east agate bearing and was clamped in the chuck which drove the rotor, and on which the measurements were made, was 0.01 mm. farther east when the rotation was right-handed than when it was left-handed.

On account of the smallness of the displacement, effect (1) is not

appreciable as shown by the measurements on the effect of axially displacing the magnetometer. Effect (2) will be much more pronounced in axial positions of the magnetometer than in equatorial positions, but can be eliminated by reversing the rotor. Effect (3) can be made inappreciable only by making the axial field gradient so small that the change in the rotor's moment due to the axial displacement does not affect the magnetometer appreciably.

In a series of experiments made before those referred to as final, effects (2) and (3) were looked for together by moving the Fe—Co rotor alternately through its maximum range east and west when the magnetometer was in an axial position, on a regular time schedule, as in the rotation experiments; but no appreciable effect was produced.

We may calculate the axial gradient necessary to produce an appreciable effect as follows: Let us assume that we have a rotor whose reversal of rotation at the speed 60 r.p.s., which is approximately equivalent to an axial field intensity of $60 \times 3.7 \times 10^{-7}$ gauss = 2.2×10^{-5} gauss, produces a deflection of 4 cm. on the magnetometer scale. Suppose the reversal of rotation produces an axial motion y cm., that the intensity gradient in the axial direction is dF/dy gauss/cm., and that the resulting deflection is d cm. Then we evidently have $d/4 = y\ dF/dy/(2.2 \times 10^{-5})$ cm. = (approx.) $0.5 \times 10^5\ y\ dF/dy$. Substituting $y = 0.001$ cm. and $d = 0.01$ cm., the minimum scale distance readable, we obtain $dF/dy = 2 \times 10^{-4}$ gauss/cm. = (approx.) $0.001\ H$/cm., which is far greater than the possible axial gradient of the residual magnetic field in the rotor's vicinity.

52. **The error from change in the rotor's azimuth or altitude.** If the (small) angle between the normal to the axis of the rotor and the component ΔF of the residual magnetic intensity lying in the meridian (practically normal to the rotor) changes by an amount $\Delta\alpha$ when the direction of rotation is reversed, the effect will be equivalent to reversing in the rotor an axial intensity $f = \frac{1}{2} \times \Delta\alpha \times \Delta F$. If we assume that the difference between the diameters of the journals and bearings is as great as 0.05 mm., the maximum possible value of $\Delta\alpha$ will be about 1/3500 radian. Hence, if we assume ΔF as great as $H/500$, the maximum possible value of f will be about 0.05×10^{-7} gauss, an intensity equivalent to rotation at a speed of about 1/70 r.p.s., and therefore negligible.

The actual clearance between the journals and the bearings in the last series of experiments varied from about 0.035 mm. to about 0.05 mm., the latter value being reached in only a few cases. In the

earlier experiments, with half-inch and eighth-inch journals, the clearance was about the same.

53. **Error from the Thomson repulsion effect.** Inasmuch as the magnetization of the rotor is never entirely symmetrical about its axis, the rotation produces an alternating magnetic field which induces currents of its own frequency in the magnetometer case and the suspension itself, and thus, on account of lack of perfect symmetry, may produce an appreciable torque on the suspension, different for different azimuths, and thus different for the two directions of rotation.

If this effect were appreciable, the values of λ in Table XIII would show a systematic dependence on the "oscillation" of column 31, which does not appear to be the case. We have, moreover, made special tests with negative results on the magnetometer with three-magnet systems. These tests were made with impressed alternating fields varying in frequency from 22 to 60 cycles per second, directed parallel, perpendicular, and at an angle of 45° to the magnetic axes, and much greater in intensity than the alternating parts of the field produced by any one of our rotors. With the intensities used, we obtained no certain permanent deflection except in the case of the most intense fields applied at 45° to the axes with the frequency 49 cycles per second. From the deflections obtained at different angles with both direct and alternating currents, from the ratio of these currents, and from the maximum angular deflection occurring on the reversal of a rotor in the principal experiments, it is easily shown that the deflection which would be produced by the Thomson effect in the principal experiments is less than about 1/300000 of the oscillation produced by very slow rotation of the rotor. It is thus in all cases entirely inappreciable.

54. **Errors from mechanical disturbances.** As the work progressed great improvements were made in the elimination of mechanical defects from the apparatus, especially within the year 1923, when we were fortunate enough to have the services of Mr. G. H. Jung on many occasions. We suspected that systematic errors might be introduced by (1) the failure of the motor to start with equal accelerations for right and left speeds, by (2) vibration and torsional shock of the rotor, and by (3) vibration of the magnetometer. Vibration of all sorts was made very small by such balancing of the rotors as was practicable and by such straightening and balancing of all but the most remote driving parts as were practicable; by bolting securely the rotor bedplate, the countershaft bearings, the motor, and the gear

box to heavy concrete piers; by keeping the motor and gear box remote from the rotor and magnetometer; and by surrounding the magnetometer supports with heavy concrete. By the use of thin countershafts near the rotor, and rubber hose connectors between the larger countershafts, torsional shock of the rotor was made very small, and there was no reason to fear a differential starting effect.

Even before the later precautions were taken the magnetometer image with well-demagnetized rotors showed no effect of the rotation (aside from displacements), except on one occasion, when it was discovered that the motor had become loose at one corner. (In some of the latest work with copper a new and poorly constructed magnetometer system had its image broadened by the rotation.)

The magnetic effect, if appreciable, of any mechanical disturbance should become manifest as a variation of rotor sensibility with speed in a compensated field. We have made experiments for the explicit purpose of testing this possibility with two rotors only. Electrolytic iron, of which one of them was made, is a substance whose residual magnetization is very easily affected by mechanical disturbances. Moreover, this particular rotor, Electrolytic Iron I, was made of a casting which had not been forged, and was quite badly unbalanced. In the experiments, which were made before the later improvements in the driving mechanism, it ran on a half-inch bronze journal and an eighth-inch stellite journal. For this rotor the rotor sensibility for the highest speed, gear ratio 2—1, was nearly 1.5% greater than for very low speed.

The other rotor, Cobalt-Nickel, had three-sixteenths inch journals of the earlier type, which were not so rigidly connected to the magnetic part of the rotor as were those of the later type. This rotor was also made of an unforged casting, but had been statically balanced. In a series of experiments at five different speeds, ranging from a few revolutions per second to about 60 r.p.s. (gear ratio 2 — 1) the rotor sensibility, as well as the absolute sensibility, which was determined with it, remained constant within the limits of the experimental error, about 1 part in 700.

55. **Errors due to inequality of right-handed and left-handed speeds.** As shown above, a number of systematic errors can be eliminated if the rotor can be driven at exactly equal speeds and with equal friction in opposite directions, conditions never exactly realized. It was therefore important to find out, for typical rotors, especially rotors for which the errors referred to are likely to be largest (those for which reversal of the magnetic axis produces the

greatest effects), how far a value of λ calculated from the mean speed for the two directions of rotation deviates from the true value when the speeds are not equal.

For this purpose resistance coils were inserted in the motor secondary for right-handed rotations, and cut out for left-handed rotations, making the right considerably slower than the left, and the value of λ calculated from the mean speed. Then the process was repeated, but with the resistances inserted for left-handed rotations.

Let T_R and T_L denote the times of 500 right- and left-handed revolutions of the motor, ΔT_1 the value of $T_R - T_L$ for the first set of observations just described, $\Delta T_2 = T_L - T_R$ for the second set, and $\Delta T = \Delta T_1 + \Delta T_2$; and let $\Delta\lambda$ denote the difference between the calculated values of λ obtained from the two sets. Then $\Delta\lambda$ is the error in λ produced by the difference of right-handed and left-handed speeds corresponding to the period difference ΔT, a quantity which was measured in nearly all of the recent experimental work.

Table VIII gives the results of all the observations which were made for the explicit purpose of studying the variations of λ and ΔT. The error $\Delta\lambda$, corresponding to the actual values of ΔT in the principal experiments, is in almost all cases entirely negligible. In the others, the proper corrections were applied.

TABLE VIII.

ERRORS DUE TO INEQUALITY OF RIGHT-HANDED AND LEFT-HANDED SPEEDS.
(Gear ratio 2–1.)

Rotor	No. sets	ΔT*	$10^7 \times \Delta\lambda$†	$10^7 \times \Delta\lambda/\Delta T$
Armco iron	4	+3.60	+0.10	+0.03
Nickel III	2	+3.12	−2.22	−0.71
	2	+2.17	−1.87	−0.86
	2	+1.30	−1.00	−0.77
		Mean for Nickel III		−0.78
Nickel I	4	+3.87	−0.08	−0.02
Preuss's alloy	2	+2.97	−0.14	−0.05

*ΔT is considered positive when right-handed rotations are slower.
†$\Delta\lambda$ is considered positive if for ΔT positive λ is greater when the positive pole of the rotor is east (and the negative pole west).

TABLE IX.

EFFECT ON λ OF ERRORS IN COMPENSATING CURRENTS.
(Gear ratio 2−1, except when otherwise stated.)

Date	Mag. pos.	Rotor	No. sets	$\Delta\lambda/\Delta V \times 10^7$	$100\Delta\lambda/\Delta C \times 10^7$	$40\Delta\lambda/\Delta B \times 10^7$
1923						
Jan. 25–26	*LEQ*	Cobalt II	27		+0.06	
Mar. 7–9	*LEQ*	Permalloy	16	+0.04 (1−1)	−0.02 (1−1 and 2−1)	
Mar. 16–17	*AN*	Steel III	18	+0.05	−0.04	
April 2, 3	*LEQ*	" "	6	−0.07	——	
May 22, 23, 28	*LEQV*	" "	12	+0.10	+0.01	−0.01
Aug. 31	*EQV*	Permalloy	2	−0.05 ± 0.03		
Sept. 1	"	"	2	−0.19 0.04		
2	*AF*	"	2	+0.02 0.10		
10	*EQV*	Steel III	2	−0.42 0.06		
11	"	" "	2	−0.21 0.04		
13	*AF*	" "	2	+0.05 0.20		
Oct. 2	*EQV*	Steel IV	2	+0.03 0.08		
16	"	Armco Fe	4		+0.08 ± 0.06	
20, 25	"	Yensen Fe (El. Fe II)	4	−0.07 0.06		
23	"	"	5		+0.08 0.06	
Nov. 4	"	Nickel III	4		+0.09 0.07	
8	"	" I	2		+0.04 0.05	
18	"	Hopk. alloy	2	−0.30 0.15		
18	"	" "	2		+0.26 0.06	
26–27	"	Cobalt II	4		+0.12 0.04	
Dec. 8	"	Yensen Fe	2	−0.20 0.07		
9	"	Heusler's alloy I	4	−0.24 0.19		
10	"	" "	2		+0.14 0.16	
13	"	Permalloy	2	+0.06 0.06		
16	"	Preuss's alloy	2		+0.01 0.05	
23–24	"	Steel IV	4		+0.03 0.02	
25	"	Steel I	3	−0.10 0.20		

56. **Errors in λ produced by errors in the compensating currents.** Table IX gives a summary of the observations made on the magnetic rotors in the course of the last two groups of experiments to determine the errors in λ due to errors in setting the compensation currents. $\Delta\lambda/\Delta V$ is the error in λ produced by increasing the current of the main compensating frame one division above the proper value; $40\Delta\lambda/\Delta B$ is the error produced by an excess of 40 divisions in the B current above the proper value; and $100\Delta\lambda/\Delta C$ is the effect on λ of a current of 100 divisions in the positive direction through the C coils. The absolute sensibilities are about the same as in the observations on copper for corresponding times (§ 35), but are not given here as they are implicit in λ.

Table X gives a summary of the errors made in setting the current V of the main compensating frame for the last series of experiments, also the values of the C current to compensate the nonuniform part of the horizontal intensity, calculated from the data of Table IV § 35, together with the actual settings of the C current for the rotation experiments. A comparison of Table X and Table IX will show that only minute, in nearly all cases negligible, errors in λ have been made by errors in setting the currents.

The actual errors in setting the V current are of course not quite so small as the table might indicate, since the current was set only to about 0.05 division.

57. **The observations for the interval September 24, 1922–August 7, 1923** (Series A). Table XI contains the results of nearly all the rotation observations made on the magnetic rotors between September 24, 1922 and August 7, 1923. At the beginning of this interval the apparatus was in much better condition than during the earlier work. The magnetometer was usually so well adjusted, and the inductors were so frequently used, that much greater errors than those actually made in setting the V and B currents could not affect the determination of λ; the rotors were, for the most part, better demagnetized and provided with new half-inch bronze and eighth-inch stellite journals very carefully worked by Mr. W. Steiner, and statically balanced; and improvements in the driving mechanism had been started by Mr. G. H. Jung. The three-phase slip-ring motor had been installed.

During the interval many improvements were made. The extra compensating coils were given almost their final disposition; the springs to keep the wires of the main compensating frame always taut

TABLE X.

ERRORS IN SETTING THE CURRENT V OF THE MAIN COMPENSATING FRAME AND THE CURRENT C OF THE C COILS.

A. Magnetometer in position EQV. Gear ratio 2–1.

Date	Rotor	Error in current V	No. days	Current C	
				Calc.	Set
				−50 (Aug. 24)	
Aug. 31–Sept. 1	Permalloy	−0.02 ± 0.00	2		−50 div.
Sept. 10–12	Steel III	0.00 0.02	3		"
Sept. 30–Oct. 4	Steel IV	−0.01 0.01	5		"
Oct. 5–8	Co-Ni	+0.04 0.03	3		"
Oct. 14–17	Armco Fe	−0.01 0.02	4		"
				−27 (Oct. 19)	"
Oct. 20–26	Yensen Fe	−0.05 0.03	6		−27
Nov. 2–5	Nickel III	−0.03 0.06	4		"
Nov. 7–10	Nickel I	−0.02 0.02	4		"
				−18 (Nov. 11)	"
Nov. 16–18	Hopk. alloy	−0.03 0.03	3		−18
Nov. 19–27	Cobalt II	−0.05 0.02	6		"
Dec. 8	Yensen Fe	−0.06	1		"
Dec. 9–12	Heusler's alloy I	−0.06 0.04	4		"
Dec. 13–15	Permalloy	−0.04 0.02	3		"
Dec. 16–18	Preuss's alloy	+0.02 0.02	3		"
				−7 (Dec. 20)	"
Dec. 22–24	Steel IV	+0.09 0.04	3		"
Dec. 25–27	Steel I	−0.04 0.01	3		"
				−12 (Dec. 28)	
Dec. 29–30	Norway Fe	+0.03 0.05	2		"
Dec. 31–Jan. 1	Steel III	+0.10 0.03	2		"
Mean error in current V		−0.01			

B. Magnetometer in position AF. Gear ratio 2–1.

				−50 (Aug. 24)	
Sept. 2–6	Permalloy	−0.07 0.01	4		−50
Sept. 13–14	Steel III	−0.06 0.00	2		−50
Mean error in current V		−0.06			

were installed; the new, heavier, and less magnetic bedplate was instituted for the old; the driving mechanism was realigned, balanced, and in general greatly improved; the magnetometer and compensator supports were rearranged so as to make the vertical equatorial positions possible, and finally imbedded in heavy concrete; and the extra compensating loops which could be thrown in and out at will, were provided to modify the degree of uniformity of the total field produced by the main compensating frame. Most of the observations were made with magnetometers with six-magnet systems, and with high sensibility. Some were made with greatly reduced sensibility, and some with the three-magnet system substituted for the six-magnet system. Most of the observations were made with lumen or babbit bearings at the east end of the rotor; lignum vitae and mahogany were substituted in a few cases.

In the table the first column gives sequence of the observations, the second the magnetometer position, the third the effect on the calculated value of λ of reversing the rotor from mark west to mark east, the fourth the number of sets in the groups, the fifth the gear ratio or speed, the sixth the mean value of λ, the seventh the average departure of the individual values in the sets, the eighth the average departure of the set means from the group mean. The symbol $2-7ls$ means that the gear ratio was $2-7$ and that a rheostat was placed in the circuit reducing the speed to a few revolutions a second, about one-third of the usual speed corresponding to the ratio $2-7$. Most of the experiments were made with the magnetometer in lower equatorial positions, some with the height increased 4.7 cm. We are not satisfied that any of these changes produced any systematic effect, and have therefore, in most cases, presented only means and their average discrepancies. Attention has been called by notes to the changes in bearings.

The most reliable results are those obtained at high speed and under such other conditions that reversal of the rotor produced only a slight effect. Those obtained with equatorial positions of the magnetometer are given in Table XII.

In Table XI results are given also for two axial positions of the magnetometer. In the position *AN* the lower system was about 12 cm. from the end of the magnetic part of the rotor; in the position *AF*, about 17 cm. The most reliable observations for *AN* agree very nearly with those for the equatorial positions, while those for *AF* give much smaller values. A number of other sets of observations were taken without the reversal of the rotors and are not included in the table, except in one or two cases where the effect of reversal was

TABLE XI.

CHIEF OBSERVATIONS ON MAGNETIC ROTORS FROM SEPTEMBER 24, 1922 TO AUGUST 7, 1923.

(Series A.)

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
Steel III — September 24, 1922 to July 14, 1923.							
1		+0.04	67	2–1	3.82	±0.06	±0.06
2		+0.05	31	1–1	4.02	0.09	0.13
3	*EQ**	+0.08	30	1–2	4.39	0.19	0.19
4		+0.20	20	2–7	4.49	02.9	0.14
5		+0.19	2	2–7*ls*	5.54	0.36	0.10
6	*AN*	−0.03	47	2–1	3.67	0.10	0.09
7	*AF*	−0.02	19	2–1	3.36	0.17	0.05
8	*AF*	−0.03	4	1–1	3.49	0.10	0.02
9	**EQ*	+0.27	3	2–1	3.61	0.10	—
10		+1.53	3	2–7	5.80	0.37	—
* Mahogany bearing E in a few cases. ** Lumen bearing W.							
(B. of S.) Electrolytic Iron I — October 22, 1922.							
1	*LEQ*	+0.22	4	2–1	4.07	±0.11	±0.05
2	*LEQ*	+0.20	3	1–1	3.40	0.16	0.06
Heusler's Alloy II — October 27–28, 1922.							
1	*LEQ**	+0.09	9	2–1	3.92	±0.16	±0.11
2	*LEQ**	+0.15	7	1–1	3.81	0.32	0.12
* Babbit bearings E and W.							
Western Electric Permalloy — December 26, 1922 to August 7, 1923.							
1		+ 0.09	34	2–1	3.73	±0.05	±0.06
2	*EQ*	+ 0.10	21	1–1	3.66	0.09	0.08
3		+ 0.04	4	1–2	3.98	0.14	—
4		+ 0.20	4	2–7	4.64	0.29	0.10
5	*AN*	+0.28	14	2–1	3.72	0.06	0.05
6	*AF*	+0.25	8	2–1	3.33	0.14	0.08
7	*AF*	−1.23	3	2–7	4.23	0.64	—

Preuss's Alloy — January 16, 1923 to April 9, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1		* +0.04	16	2–1	3.63	±0.07	±0.08
2		+0.32	20	1–1	3.72	0.18	0.13
3		+0.29	3	2–1	3.79	0.07	—
4	*LEQ*	+0.87	4	1–1	4.13	0.10	0.09
5		** +1.03	3	1–2	4.60	0.12	—
6		+0.51	3	2–7	5.32	0.29	—
7		−0.09	3	2–7*ls*	5.98	1.31	—
		* Wood bearing E. ** Lumen bearing E.					
		Cobalt II — January 25–30, 1923.					
1	*LEQ*	−0.005	16	2–1	3.79	±0.14	±0.08
2	*LEQ*	−0.16	17	1–1	3.68	0.22	0.20
		Armco Iron — June 17, 1923.					
1	*LEQ*	+0.11	4	2–1	3.99	±0.05	±0.02
2	*LEQ*	−0.07	2	2–7	3.20	0.23	—
		Steel II — January 16–23, 1923.					
1	*LEQ*	+0.03	4	2–1	3.72	±0.07	±0.04
2	*LEQ*	+0.08	2	2–7	3.14	0.34	—
3	*LEQ*	−0.06	4	2–1	3.84	0.08	—
4	*LEQ*	* −0.26	4	1–1	3.64	0.20	—
5	*LEQ*	+0.35	2	1–2	3.18	0.29	—
6	*LEQ*	+1.10	2	2–7	3.51	0.51	—
		* Lumen half-inch and bronze eighth-inch bearings W.					

approximately known. These omitted observations agree well with the others.

For most of the rotors the values of λ for the gear ratio 1−1 do not differ greatly from those for 2−1. But in general for the lower speeds λ increases or decreases considerably, or even very greatly, as the speed goes down. For these speeds the accidental errors are relatively quite large, but the systematic effects are nevertheless undoubtedly authentic.

With the precautions taken and the tests made with reference to eddy-current effects we could not see how these discrepancies could be attributed to them. Also, some of the rotors were so well demagnetized and had journals at the two ends so nearly identical, and the right and left speeds were so nearly equal, that it seemed difficult to attribute the discrepancies to journal or bearing effects. We nevertheless became convinced that such effects were chiefly responsible, and therefore proceeded to modify the rotors and bearings in such a way as to cut down the heat developed at the bearings, at least the east bearing, and to insulate the journals from both the magnetic part of the rotor and from the bearing blocks. The devices used have already been described.

TABLE XII.

λ FROM THE BEST EQUATORIAL OBSERVATIONS OF TABLE XI.

A. Gear ratio 2–1.

Rotor	Group	$10^7 \times \Delta\lambda$(E–W)	$-\lambda \times 10^7$	$\Delta\lambda'' \times 10^7$	No. sets
Steel III	1	+0.04	3.82	±0.06	67
Heusler's alloy II	1	+0.09	3.92	0.11	9
Permalloy	1	+0.09	3.73	0.06	34
Preuss's alloy	1	+0.04	3.63	0.08	16
Cobalt II	1	−0.05	3.79	0.08	16
Armco iron	1	+0.11	3.99	0.02	4
Steel II	1 and 3	−0.02	3.78	0.04±	4
Weighted mean $\lambda \times 10^7$ for 150 sets = −3.78 ± 0.06					Total 150

B. Gear ratio 1–1.

Rotor	Group	$10^7 \times \Delta\lambda$(E–W)	$-\lambda \times 10^7$	$\Delta\lambda'' \times 10^7$	No. sets
Steel III	2	+0.05	4.02	±0.13	31
Heusler's alloy II	2	+0.15	3.81	0.32	7
Permalloy	2	+0.10	3.66	0.08	21
Preuss's alloy	2	+0.32	3.72	0.13	20
Cobalt II	2	−0.16	3.68	0.20	17
Steel II	4	−0.26	3.64	—	4
Weighted mean $\lambda \times 10^7$ for 100 sets = − 3.80 ± 0.15					Total 100

58. **The last series of observations on** λ (Series B). The last series was begun near the end of August, 1923, and was completed on January 1, 1924.

A few observations, on Steel III and Permalloy, were made with the magnetometer in the position *AF*, the remainder with the magnetometer in the position *EQV*, as in the later observations of the series immediately preceding.

For these experiments all the rotors to be used were provided with three-sixteenths inch insulated journals of one of the two types already described in § 12 and illustrated in Figure 16, (a) and (b). The earlier experiments in the series were made with journals of type (a), the remainder with journals of type (b), which were mechanically much superior. The earlier experiments with Permalloy were made with bakelite bearings, all the others with agate bearings. Except in the case of the rotor of Bloch's alloy (Co-Ni) all of those rotors which were at first provided with journals of type (a) were later provided with, and rotated with, journals of type (b).

The substitution of these small, insulated, and almost perfectly made journals, together with the agate bearings, for those used before, the correction of the warp in the main compensating frame, the perfecting of the method of adjusting the compensator to correct for imperfections in the magnetometer, and the insertion of a cardboard screen between rotor and bedplate in part of the work or the surrounding of the rotor with a uniform cylindrical cardboard tube in much of the remainder of the work, constituted the chief sources of improvement over the conditions in which the immediately preceding series of observations was finished.

Unfortunately, the mirror of the excellent six-magnet magnetometer system with which the series was begun was not securely fastened, and became displaced while the magnetometer was being moved from one position to the other. In attempting to remount the mirror in better fashion the system was injured, and many attempts to put it in first-class condition again were failures. It was used again for observations on copper near the end of the series; but after the first month's work the three-magnet system already mentioned, and constructed only for special tests, was used exclusively with the magnetic rotors. Inasmuch as we were continually expecting to be able to use the six-magnet system again, this state of things led to the magnetometer's azimuth being usually not so well adjusted as in most of the preceding series; but, as the observations on V and $\Delta\lambda/\Delta V$, Tables IX and X, show, this introduced no error of any consequence. The three-magnet

system was considerably slower than the six-magnet systems, but not too slow to avoid appreciable error, except possibly with those rotors (Heusler's alloy and Preuss's alloy) which exhibit considerable magnetic lag. We estimate that an error of one-fourth per cent is possible with these rotors.

During most of the immediately preceding series of observations induction measurements for setting the V and B currents were made almost every night, so that, even had the variometers been very unreliable, eddy-current errors from incorrect settings of these currents could have exceeded only very small quantities. In the last series, however, the severity of the work necessitated limiting greatly the number of induction measurements. This procedure was adopted with less reluctance because the earlier work had shown that, with good magnetometer adjustment, very considerable errors in the current settings could be made without affecting the values of λ appreciably. The observations in Tables IX and X show that only negligible errors in λ could have arisen from errors in setting the V and C currents, at least in nearly all cases. Had the C current been made zero, as the work with the induction balance seems to indicate was correct, the values of λ would have been changed (*increased*) by only minute amounts.

Throughout the last series the V and B current settings were made from the limited number of induction observations and the variometer readings. The change in the V current was so slight and so slow that it introduced no difficulty. Our observations have been confirmed by the vertical intensity observations of the Cheltenham Observatory, which show that this intensity decreased very slowly, by only a few parts in 5000, from the beginning to the end of our work. On one night when the work was in progress it was low by about 1 part in 1000.

Unfortunately, changes, slight as they were, in the compensating frame had greater effects on the proper value of the current B. As the exact effects of these changes were unknown, it was impossible, from the relatively few induction determinations and the variometer readings taken twice every night, to predetermine with exactness the proper value of B. No direct observations for $\Delta\lambda/\Delta B$ were made. However, from the Cheltenham horizontal intensity observations, which we have used generally in preference to our own less complete observations (although these appear to be reliable), and interpolation and extrapolation (after December 14) from our induction determinations, on the assumption of linear changes in the frame with time, it was possible to calculate for each night, or each hour of each night, the

correct value of the current B. The amount ΔB by which our current B exceeded the correct value, obtained in the way indicated, is given for each group of observations in column 33 of Table XIII. Combined with the results of Table V, § 36, these values of ΔB show, as indicated in § 36, that no appreciable errors originated from the departure of ΔB from zero.

59. **Example of a night's work.** As an example of one night's work, neither as good as the best nor as bad as the worst, we shall give the results obtained on December 13, 1923, those for the first part of the night in detail, those for the latter part by means. The rotor was Permalloy, running in agate bearings and inclosed in the protecting cylinder of pasteboard, one side, X, being up during the first part of the work, and down the rest. The magnetometer was in the position EQV. The times are only approximately correct, and refer to the beginnings of the different groups.

$2^h\ 24^m$ A.M.— VARIOMETER READINGS AND CURRENT SETTINGS.

	Horizontal	Vertical	Declination
Temp.	19°.0 C.	18°.8 C.	——
Scale	+2.06 cm.	+0.46 cm.	56.95 div.
Comp. current	+0.79 amp. (B)	1.384+amp. (V)	1.49 amp. (A)

The C current was set at −0.18 amp.

$2^h\ 33^m$ A.M.— ABSOLUTE SENSIBILITY SCALE READINGS.

Switch R	Switch L	$A = \lvert R - L \rvert$
−1.48 cm.		
	+16.81 cm.	18.26 cm.
1.31		.17
	16.91	.15
1.16		.12
	17.00	.18
1.20		.16
	16.91	.01
0.99		.02
	17.19	

Mean $A = 18.13$ cm. ± 0.06 cm.

ROTATIONS. MARK E. X UP.

$2^h\ 50^m$ A.M.— SCALE READINGS.

Set 1 Gear ratio 2−1 $V = 138.4+$

R	L	$R - L = d$	Δd	T_R (Time of 500 rev.)	T_L
+10.60 cm.				$16^s.90$	
	+6.30 cm.	4.35 cm.	−0.01 cm.		$16^s.90$
10.70		4.35	− .01	.63	
	6.40	4.35	− .01		.80
10.80		4.35	− .01	.52	
	6.50	4.41	+ .05		.67
11.02		4.47	+ .11	.52	
	6.60	4.46	+ .10		.58
11.10		4.32	− .04	.48	
	6.97	4.23	− .13		.52
11.30		4.26	− .10	.50	
	7.10				.54
	Means	4.36	±0.06		

Mean $T_R = 16.59$ $T_L = 16.67$

Mean $T = 16.63$

$T_R - T_L = -\ 0.08$

$3^h\ 8^m$ A.M.— SCALE READINGS.

Set 2 Gear ratio 2−7 $V = 138.4+$

R	L	R − L = d	Δd	T_R	T_L
+10.29 cm.				$16^s.08$	
	+9.47 cm.	0.675 cm.	+0.06 cm.		$16^s.02$
10.20		0.565	−0.05	.04	
	9.80	0.51	−0.10		.02
10.42		0.565	−0.05	.03	
	9.91	0.65	+0.04		.02
10.70−		0.74	+0.13	.05	
	10.01	0.79	+0.18		.04
10.90		0.76	+0.15	.02	
	10.27				.02
——		0.65	+0.04	.04	
	10.30				.02
10.97		0.51	−0.10	.00	
	10.62	0.42	−0.19		.07
11.11		0.59	−0.02	.02	
	10.41+	0.74	+0.13		.07
11.20−		0.665	+0.05	.05	
	10.66	0.645	+0.03		.04
11.41		0.62	+0.01	.05	
	10.92+	0.54	−0.07		.03
11.52		0.56	−0.05	——	
	11.00	0.56	−0.05		——
11.60		0.50	−0.11	——	
	11.20				
	Means	0.613	±0.08	Means 16.04	16.06
$V = 138.3$				Mean T 16.05	
				$T_R - T_L$ −0.02	

MARK E. X DOWN.

Time	Set	Gear ratio	d	Δd	T	$T_R - T_L$	V
$3^h 42^m$ A.M.	3	2−1	4.46	±0.07	16.52	−0.12	1.389−
58	4	2−1	4.41	0.08	16.46	−0.06	1.379−
4 16	5	2−7	0.623	0.10	16.00	+0.07	1.384

$4^h\ 37^m$ A.M.— ABSOLUTE SENSIBILITY.

$A = 18.24$

E.m.f. of battery = 1.546 volt

Ammeter — potentiometer = + 0.0015 amp.

$4^h\ 52^m$ A.M.— VARIOMETER AND CURRENT READINGS.

Horizontal	Vertical	Declination
Temp. 19°.5 C.	19°.1 C.	——
Scale +1.91 cm.	+0.44 cm.	56.2+
Comp. current 0.785 amp. (B)	1.384 amp. (V)	1.43 amp. (A)
The C coil current read −0.18 amp.		

From the above variometer scale and temperature readings, together with the variometer constants given in § 12, we find that the horizontal and vertical intensities at the end of the night differed from the intensities at the beginning by only about 1 part in 11,000 and 1 part in 34,000, respectively. The declination changed by only 1 minute.

The value of $Q = B/A$, determined on the preceding day, was $Q = 2.936$; the value of K was 0.0%.

From what precedes, and the formula (8) of § 25, we obtain the following values of λ:

Set	Gear ratio		$-\lambda \times 10^7$		$\Delta\lambda \times 10^7$
1	2−1		3.75		±0.05
2	2−7		3.56		0.46
3	2−1	over and under	3.79	3.76	0.06
4	2−1	compensated	3.73		0.07
5	2−7		3.59		0.58

which are to be combined with the observations of the next day, with the rotor reversed, to obtain values of λ more nearly free from error. The final results are given in Table XIII.

60. **The results of the last series** (Series B). This table (Table XIII) gives a summary of the results for the whole series. The values of λ are given in columns 6, 12, 18, and 24. Columns 4, 10, 16, 22 give the number of sets in each group of observations. Columns 7, 13, 19, and 25 give the average departure from the mean in the separate sets of which the groups are composed, while columns 8, 14, 20, and 26 give the average departure, from the mean for each group, of the means for the individual sets of which it is composed. Columns 3, 9, 15, 21 give ΔT, which is the excess of T for right-handed rotations over T for left-handed rotations when the marked end of the rotor is east *minus* the similar quantity when the marked end is west. Columns 5, 11, 17, 23 give the amounts by which the experimental value of λ with mark east exceeds that with mark west. It is twice the error

1 Date	2 Rotor	Gear ratio: 2−7 3 ΔT	4 No. sets	5 $10^7 \times \Delta\lambda$ (E − W)	6 $-\lambda \times 10^7$	7 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	8 Av. dep. (sets) $10^7 \times \Delta''\lambda$	9 ΔT
Oct. 20–21	El. iron II	$-0^s.02$	3	+1.06	3.95	±0.49	—	
Oct. 23–26		+0.05	2	−0.05	3.84	0.48	—	+0.05
Dec. 8								
Oct. 14	Armco iron	—	2	+0.21	3.60	0.44	±0.16	+0.06
Dec. 29–30	Norway iron	−0.03	5	−0.37	2.63	0.86	0.36	
Sept. 10–12	Steel III ¶	—	4	−0.31	4.07	0.35	0.32	—
Sept. 13	*AF* ¶	—	2	east only	4.85	0.60	0.19	—
Oct. 3, 4	¶	—	2	west only	3.86	0.32	0.08	
Dec. 31–Jan. 1		+0.01	5	−0.15	3.71	0.37	0.11	
Sept. 30–Oct. 2	Steel IV ¶	+0.08	2	−0.36	4.41	0.30	0.18	0.00
Dec. 22–24		−0.02	6	−0.46	3.65	0.49	0.25	
Dec. 25–27	Steel I	−0.11	4	+0.28	3.60	0.57	0.19	
Nov. 2–3	Nickel III							+0.02
Nov. 4–5								
Nov. 8–10	Nickel I	+0.01	2	+0.28	3.49	0.70	0.14	+0.03
Nov. 19–21	Cobalt II							
Nov. 22–27								−0.01
Dec. 9–12	Heusler's alloy I	−0.01	4	+0.41	3.60	0.77	0.24	−0.07
Aug. 31–Sept. 1	Permalloy ¶	—	5	+0.08	3.97	0.26	0.15	—
Sept. 2–6	*AF* ¶							+0.02
Dec. 13–14		+0.02	3	−0.42	3.78	0.60	—	
Nov. 16–18	Hopkinson's alloy							
Dec. 16–18	Preuss's alloy	+0.01	4	−0.72	4.48	0.55	0.36	−0.02
Oct. 5–8	¶ Cobalt-nickel	+0.03	5	+0.07	3.77	0.60	0.43±	+0.07

¶ Rotor provided with earlier style three-sixteenths inch journals.

Means: 3.73 ± 0.51 ± 0.25.

Mean T ranged from $15^s.90$ to $16^s.11$.

Total number of sets = 60. In obtaining the mean value of λ only those sets (56) were used which occurred in groups in which the rotor was reversed.

Mean
Mean
Total

TABLE XIII.

SUMMARY OF OBSERVATIONS AND RESULTS FOR FERROMAGNETIC ROTORS

1–2						1–1					
9 ΔT	10 No. sets	11 $10^7 \times \Delta\lambda$ (E − W)	12 $-\lambda \times 10^7$	13 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	14 Av. dep. (sets) $10^7 \times \Delta''\lambda$	15 ΔT	16 No. sets	17 $10^7 \times \Delta\lambda$ (E − W)	18 $-\lambda \times 10^7$	19 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	20 Av. dep. (sets) $10^7 \times \Delta''\lambda$
+0.05	2	−0.06	4.01	±0.35	—	−0s.02	4	−0.01	3.98	±0.17	±0.01
+0.06	2	0.15	3.60	0.24	±0.08	−0.03	2	+0.03	3.60	0.11	0.02
—	3	+0.11	3.76	0.24	0.11	—	4	+0.01	3.86	0.14	0.24
—	1	east only	3.93	0.26	—	—	2	east only	3.75	0.24	0.26
						−0.01	5	+0.06	3.83	0.07	0.04
0.00	2	+0.04	4.10	0.28	0.02	+0.02	2	−0.04	4.11	0.08	0.02
						0.00	4	+0.23	3.72	0.16	0.07
+0.02	2	−1.27	2.86	0.35	—	+0.02	4	−0.38	3.59	0.18	0.04
+0.03	2	+0.30	3.74	0.36	0.15	+0.12	2	0.00	3.67	0.21	0.00
−0.01	6	−0.02	3.85	0.45	0.17	−0.04	6	0.00	3.73	0.17	0.14
−0.07	4	+1.10	3.83	0.54	0.26	0.00	4	+0.01	3.48	0.24	0.12
—	3	+0.34	3.87	0.15	0.17	—	3	+0.04	3.76	0.08	0.05
+0.02	6	−0.08	3.90	0.58	0.03	+0.01	4	+0.03	3.91	0.26	0.04
						0.00	8	+0.01	3.87	0.36	0.22
−0.02	4	+0.16	4.06	0.22	0.10	+0.05	4	+0.01	3.83	0.17	0.06
+0.07	4	+0.13	3.59	0.28	0.12	0.00	4	+0.09	3.81	0.12	0.05

Means: 3.81 ± 0.36 ± 0.12.
Mean T ranged from 16s.02 to 16s.24.
Total number of sets = 41. In obtaining the mean value of λ only those sets (34) were used which occurred in groups in which the rotor was reversed.

Means: 3.79 ± 0.19 ± 0.10.
Mean T ranged from 16s.13 to 16s.36.
Total number of sets = 62.

2−1

21 ΔT	22 No. sets	23 $10^7 \times \Delta\lambda$ (E − W)	24 $-\lambda \times 10^7$	25 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	26 Av. dep. (sets) $10^7 \times \Delta''\lambda$	27 Approx. d (2−1)	28 Approx. A	29 Q	30 $I \times 10^8$ Begin.	30 $I \times 10^8$ End	31 Osc.	32 ΔO (2−1)	33 ΔE
						cm.	cm.				cm.	cm.	div.
$-0^s.03$	4	+0.03	3.97	±0.08	±0.05	3.6−3.8	18.4−19.2	2.337	*+12.5	+19.2	5.8	0.3	− 4
+0.03	9	−0.07	3.92	0.06	0.05	3.7−3.8	18.9−19.2	2.337	*+ 5.1	− 0.1	1.5	0.4	+ 3
−0.05	5	0.00	3.83	0.08	0.02	3.5−3.6	17.9−18.0	2.379	*+ 1.9	0.0	1.5	0.0	+ 5
−0.05	10	0.00	3.67	0.06	0.04	2.3−2.5	18.0−18.8	1.684	*− 3.5	− 1.0	4.5	0.2	+ 2
−0.01	6	−0.08	3.69	0.12	0.04	2.6	17.2−17.4	1.885	*− 0.9	+ 0.1	26±	0.6	+18
—	6	+0.01	3.76	0.06	0.03	3.6−3.9	19.0−20.4	2.355	*− 2.1	− 4.0	4.5	0.9	− 0
—	2	east only	3.81	0.20	—	1.7	19.4	1.104	*− 4.0	− 5.2	1.2	0.1	
+0.01	5	0.00	3.77	0.05	0.05	3.5−3.6	17.9−18.7	2.371	*− 5.2	− 5.7	2.5	0.4	+ 1
+0.05	4	−0.16	3.72	0.06	0.02	3.1−3.2	16.1−16.7	2.371	*+ 1.3	+ 5.0	1.0	0.6	+ 9
+0.04	8	+0.06	3.94	0.07	0.03	3.0−3.3	17.6−19.0	1.973	*+ 7.8	+ 2.0	8.7	0.6	− 1
−0.02	13	−0.05	3.73	0.06	0.06	3.0−3.1	17.9−18.8	2.026	*− 8.2	−10.4	0.7	0.4	+10
−0.02	9	−0.11	3.75	0.10	0.07	2.3−2.6	15.4−18.1	1.802	− 1.7	− 0.1	2.4	0.7	+13
−0.06	4	−0.14	3.67	0.09	0.01	3.2−3.3	18.4	2.199	− 5.8	− 4.4	0.9	6.6	+ 4
+0.03	6	−0.56	3.60	0.08	0.03	3.4−3.5	19.1	2.199	*− 4.4	− 4.9		—	+ 6
−0.03	9	0.00	3.75	0.08	0.06	2.3−2.5	17.2−17.5	1.752	*+ 2.1	2.1	6.0	0.1	+10
−0.02	6	−0.07	3.86	0.09	0.06	2.8−2.9	17.2−18.0	1.937	*+11.3	+19.7	3.8	0.1	+ 6
−0.03	7	−0.05	3.82	0.10	0.05	2.9−3.0	17.9−18.3	1.943	*− 1.1	− 5.5	8.7-6.1	0.4	+ 5
−0.01	10	+0.05	3.62	0.20	0.06	1.5−1.6	15.9−18.4	1.145	*+ 3.2	+ 4.4	5.8-6.2	1.0	+ 7
—	4	−0.02	3.77	0.04	0.06	4.7−4.9	19.3−19.7	2.960	*− 2.2	—	8.8	0.7	− 6
−0.06	7	+0.12	3.84	0.14	0.03	2.1−2.3	17.7−19.7	1.386	− 6	.0	3.0	0.3	
+0.05	5	+0.10	3.70	0.06	0.05	4.2−4.5	18.0−18.2	2.936	*+ 0.6	+ 0.3	8.8	0.7	+ 8
−0.04	10	+0.04	3.63	0.18	0.06	1.2−1.5	15.7−18.4	1.047	+ 7.3	+13.3	14.4	0.7	+ 5
−0.12	6	+0.04	3.83	0.09	0.02	3.8−4.0	17.6−18.5	2.585	*+ 0.1	− 1.7	5.9	0.4	+11
+0.08	4	0.005	3.83	0.06	0.04	2.5−2.6	18.5−18.9	1.651	*+ 1.1	+ 4.3	9.0	0.2	+ 4

Means: 3.76 ± 0.09 ± 0.05.
Mean T ranged from $16^s.30$ to $16^s.64$.
Total number of sets = 159.

* Observation of I made on day preceding that in wh

TABLE XIII.

NS AND RESULTS FOR FERROMAGNETIC ROTORS FROM AUGUST 31, 1923 TO JANUARY 1, 1924.

1−1					2−1												
	16 No. sets	17 $10^7 \times \Delta\lambda$ (E−W)	18 $-\lambda \times 10^7$	19 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	20 Av. dep. (sets) $10^7 \times \Delta''\lambda$	21 ΔT	22 No. sets	23 $10^7 \times \Delta\lambda$ (E−W)	24 $-\lambda \times 10^7$	25 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	26 Av. dep. (sets) $10^7 \times \Delta''\lambda$	27 Approx. d (2−1)	28 Approx. A	29 Q	30 $I \times 10^3$ Begin.	30 $I \times 10^3$ End	31 Osc.	32 ΔQ (2−1)	
												cm.	cm.				cm.	cm.	c
2	4	−0.01	3.98	±0.17	±0.01	−0^s.03	4	+0.03	3.97	±0.08	±0.05	3.6−3.8	18.4−19.2	2.337	*+12.5	+19.2	5.8	0.3	−
						+0.03	9	−0.07	3.92	0.06	0.05	3.7−3.8	18.9−19.2	2.337	*+ 5.1	− 0.1	1.5	0.4	+
						−0.05	5	0.00	3.83	0.08	0.02	3.5−3.6	17.9−18.0	2.379	*+ 1.9	0.0	1.5	0.0	+
3	2	+0.03	3.60	0.11	0.02	−0.05	10	0.00	3.67	0.06	0.04	2.3−2.5	18.0−18.8	1.684	*− 3.5	− 1.0	4.5	0.2	+
						−0.01	6	−0.08	3.69	0.12	0.04	2.6	17.2−17.4	1.885	*− 0.9	+ 0.1	26±	0.6	+
	4	+0.01	3.86	0.14	0.24	—	6	+0.01	3.76	0.06	0.03	3.6−3.9	19.0−20.4	2.355	*− 2.1	− 4.0	4.5	0.9	−
	2	east only	3.75	0.24	0.26	—	2	east only	3.81	0.20	—	1.7	19.4	1.104	*− 4.0	− 5.2	1.2	0.1	
1	5	+0.06	3.83	0.07	0.04	+0.01	5	0.00	3.77	0.05	0.05	3.5−3.6	17.9−18.7	2.371	*− 5.2	− 5.7	2.5	0.4	+
						+0.05	4	−0.16	3.72	0.06	0.02	3.1−3.2	16.1−16.7	2.371	*+ 1.3	+ 5.0	1.0	0.6	+
2	2	−0.04	4.11	0.08	0.02	+0.04	8	+0.06	3.94	0.07	0.03	3.0−3.3	17.6−19.0	1.973	*+ 7.8	+ 2.0	8.7	0.6	−
						−0.02	13	−0.05	3.73	0.06	0.06	3.0−3.1	17.9−18.8	2.026	*− 8.2	−10.4	0.7	0.4	+
0	4	+0.23	3.72	0.16	0.07	−0.02	9	−0.11	3.75	0.10	0.07	2.3−2.6	15.4−18.1	1.802	− 1.7	− 0.1	2.4	0.7	+
2	4	−0.38	3.59	0.18	0.04	−0.06	4	−0.14	3.67	0.09	0.01	3.2−3.3	18.4	2.199	− 5.8	− 4.4	0.9	6.6	+
						+0.03	6	−0.56	3.60	0.08	0.03	3.4−3.5	19.1	2.199	*− 4.4	− 4.9		—	+
2	2	0.00	3.67	0.21	0.00	−0.03	9	0.00	3.75	0.08	0.06	2.3−2.5	17.2−17.5	1.752	*+ 2.1	2.1	6.0	0.1	+
						−0.02	6	−0.07	3.86	0.09	0.06	2.8−2.9	17.2−18.0	1.937	*+11.3	+19.7	3.8	0.1	+
4	6	0.00	3.73	0.17	0.14	−0.03	7	−0.05	3.82	0.10	0.05	2.9−3.0	17.9−18.3	1.943	*− 1.1	− 5.5	8.7-6.1	0.4	+
0	4	+0.01	3.48	0.24	0.12	−0.01	10	+0.05	3.62	0.20	0.06	1.5−1.6	15.9−18.4	1.145	*+ 3.2	+ 4.4	5.8-6.2	1.0	+
	3	+0.04	3.76	0.08	0.05	—	4	−0.02	3.77	0.04	0.06	4.7−4.9	19.3−19.7	2.960	*− 2.2	—	8.8	0.7	−
1	4	+0.03	3.91	0.26	0.04	−0.06	7	+0.12	3.84	0.14	0.03	2.1−2.3	17.7−19.7	1.386	− 6	.0	3.0	0.3	
						+0.05	5	+0.10	3.70	0.06	0.05	4.2−4.5	18.0−18.2	2.936	*+ 0.6	+ 0.3	8.8	0.7	+
0	8	+0.01	3.87	0.36	0.22	−0.04	10	+0.04	3.63	0.18	0.06	1.2−1.5	15.7−18.4	1.047	+ 7.3	+13.3	14.4	0.7	+
5	4	+0.01	3.83	0.17	0.06	−0.12	6	+0.04	3.83	0.09	0.02	3.8−4.0	17.6−18.5	2.585	*+ 0.1	− 1.7	5.9	0.4	+
0	4	+0.09	3.81	0.12	0.05	+0.08	4	0.005	3.83	0.06	0.04	2.5−2.6	18.5−18.9	1.651	*+ 1.1	+ 4.3	9.0	0.2	+

ans: 3.79 ± 0.19 ± 0.10.
an T ranged from 16^s.13 to 16^s.36.
tal number of sets = 62.

Means: 3.76 ± 0.09 ± 0.05.
Mean T ranged from 16^s.30 to 16^s.64.
Total number of sets = 159.

* Observation of I made on day preceding that in

33 ΔB	34 Remarks
div.	
− 4.0	
+ 3.4	
+ 5.5	Tube on. (See §42.)
+ 2.6	
+18.0	Tube on.
− 0.2	Six-magnet systems.
	Six-magnet systems.
+ 1.0	
+ 9.3	Tube on. New journals.
− 1.7	
+10.8	Tube on. New journals.
+13.6	Tube on.
+ 4.2	Card on.
+ 6.2	Bag on and card. (See §49.)
+10.0	Card on.
+ 6.6	Tube on.
+ 5.0	Tube on.
+ 7.8	Tube on.
− 6.8	Six-magnet systems. Bakelite bearings.
	Bakelite bearings. *AF*
+ 8.6	Agate bearings. New journals. Tube on.
+ 5.9	Tube on.
+11.7	Tube on.
+ 4.2	

t in which rotations began.

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which would be made if the rotor were not reversed. Column 27 gives the approximate value of the deflection d produced on reversing the rotor with the gear ratio $2 - 1$ (speed approximately 60 r.p.s.); column 28, the approximate value of the absolute sensibility A; column 29, the exact value of the ratio Q of rotor sensibility to absolute sensibility; column 30, the approximate values of the mean intensity of magnetization of the rotor in c.g.s. units; column 31, the approximate range of motion of the spot of light on the magnetometer scale when the rotor is turned through a complete revolution with extreme slowness; column 32, the approximate amount by which the magnetometer zero is changed when the speed changes from a few revolutions per second to the highest speed (except that for Nickel III, these quantities, obtained from rough observations, may all be spurious, as many of them doubtless are). Column 33 gives the error ΔB in the B current, as has already been explained.

The value of λ in each group in which the rotor was reversed was obtained by giving equal weights to the observations for mark east and mark west, usually equal or nearly equal in number. The means were obtained by giving each group a weight proportional to the number of sets it contains.

It is clear from the table that the discrepancy between the values of λ for the positions AF and EQV no longer exists; also, that in the mean no variation of λ with speed is apparent. This latter fact alone is strong evidence that the chief systematic errors are eliminated, as most of them produce deflections which are nearly independent of the speed, or proportional to the square of the speed, while the deflection produced by the effect under investigation is proportional to the speed.

The most reliable results are those obtained with the gear ratio $2-1$ and are collected in Table XIV. The last column gives the numerical departures $\Delta'''\lambda$ of the values of λ for the individual rotors from the mean for all the rotors, and the mean value of these departures. $\Delta'\lambda$ and $\Delta''\lambda$ have the same meanings as in Table XIII. All quantities are given weights proportional to the number of sets of observations involved. Although $\Delta'''\lambda$ is nearly twice as great as $\Delta''\lambda$, an examination of the values of λ obtained with different rotors of the same or nearly the same material, and in different groups of observations (Table XIV) on the same rotors, reveals discrepancies making uncertain any conclusion that real differences in λ for the different rotors and materials exist.

If the values of λ for all the rotors in Table XIV are given equal weights, we obtain the mean 3.75 in place of 3.76.

TABLE XIV.

λ FROM OBSERVATIONS AT THE HIGHEST SPEED.

Rotor	No. sets	$-\lambda \times 10^7$	$\Delta'\lambda \times 10^7$	$\Delta''\lambda \times 10^7$	$\Delta'''\lambda \times 10^7$
Yensen iron	18	3.91	±0.07	±0.04	0.15
Armco iron	10	3.67	0.06	0.04	0.09
Norway iron	6	3.69	0.12	0.04	0.07
Steel III	17	3.76	0.07	0.03	0.00
Steel IV	21	3.81	0.06	0.05	0.05
Steel I	9	3.75	0.10	0.07	0.01
Mean for iron and steel (6 rotors)	(81)	3.79	0.07	0.04	0.06
Nickel III	10	3.64	0.08	0.02	0.12
Nickel I	9	3.75	0.08	0.06	0.01
Mean for nickel (2 rotors)	(19)	3.69	0.08	0.04	0.05+
Cobalt II	13	3.84	0.10	0.05	0.08
Heusler's alloy I	10	3.62	0.20	0.06	0.14
Permalloy	16	3.78	0.09	0.04	0.02
Hopkinson's alloy	10	3.63	0.18	0.06	0.13
Preuss's alloy (Fe−Co)	6	3.83	0.09	0.02	0.07
Bloch's alloy (Co−Ni)	4	3.83	0.06	0.04	0.07
Mean for 159 sets	=	3.76	±0.09	±0.05	±0.07
Mean $\lambda/2\pi$	=	$1.06 \times m/e \pm 2.4\%$		± 1.2%	±1.9%

Almost exactly the same mean was obtained from the experiments of the immediately preceding series (Table XII), but the work done in the final series has increased the degree of certainty of the results very greatly.

It is a source of great satisfaction to us that our early work by the method of electromagnetic induction, including the earliest work, of 1914, agrees so closely with all of our recent determinations. The freedom of this early work from systematic errors is due largely to the great symmetry of the experimental arrangement for detecting the changes of magnetization produced by the rotation.

61. **Recent experiments on rotation by magnetization.** We have already referred to the effect converse to ours and discovered in the following year by Einstein and de Haas, viz., rotation by magnetization. A considerable amount of work, some of which has already been mentioned, has now been done on this effect. The most thorough investigations are those of Emil Beck,[33] who used a method of resonance greatly improved over the original method used by Einstein and de Haas; Chattock and Bates,[34] who used a ballistic method improved over that of J. Q. Stewart; and Sucksmith and Bates,[35] who used a null resonance method.

On the simplest classical theory and the assumption of a single simple magneton, these experiments determine the ratio R of its angular momentum to its magnetic moment. On the same theory, as already stated, this value of R should agree with our value of $\lambda/2\pi$; and both should be equal to $2\ m/e$ for a Bohr magneton.

All the most reliable of these experiments, as a matter of fact, give values for R not far from half this quantity, viz., m/e, like our own value of $\lambda/2\pi$.

The mean value found by Beck for iron and nickel is $1.10 \times m/e$; that found by Chattock and Bates, for the same metals, is $1.00 \times m/e$, which they consider correct within less than 1%. The discrepancies between the individual observations from which their final means are derived, however, are so large that we do not feel confident their error is so small. For iron, nickel, and Heusler's alloy, Sucksmith and Bates have found $1.00 \times m/e$, with an apparent error of about 1%.

The mean value of $\lambda/2\pi$ obtained from our rotation experiments, as given in Table XIV, is $1.06 \times m/e$. We do not see how the error can be greater than 2%.

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[33] *Ann. d. Phys.*, **60**, 109, 1919; see also G. Arvidsson, *Phys. Zeit.*, **21**, 88, 1920.

[34] *Roy. Soc. Phil. Trans. A*, **223**, 257, 1923.

[35] *Roy. Soc. Proc. A*, **104**, 499, 1923.

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